# MATHEMATICS 7

## Statistics and Probability

Distance Learning

Alberta EDUCATION

## Module 7

Mathematics 7

Module 7

# STATISTICS AND PROBABILITY


Alberta
EDUCATION

Mathematics 7
Student Module Booklet
Module 7
Statistics and Probability
Alberta Distance Learning Centre
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| This document is intended for | |
|---|---|
| Students | ✓ |
| Teachers (Mathematics 7) | ✓ |
| Administrators | |
| Parents | |
| General Public | |
| Other | |


The Alberta Distance Learning Centre has an Internet site that you may find useful. The address is http://ednet.edc.gov.ab.ca/level3/adlc/300/adlchome.html.

The use of the Internet is optional. Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

# Welcome

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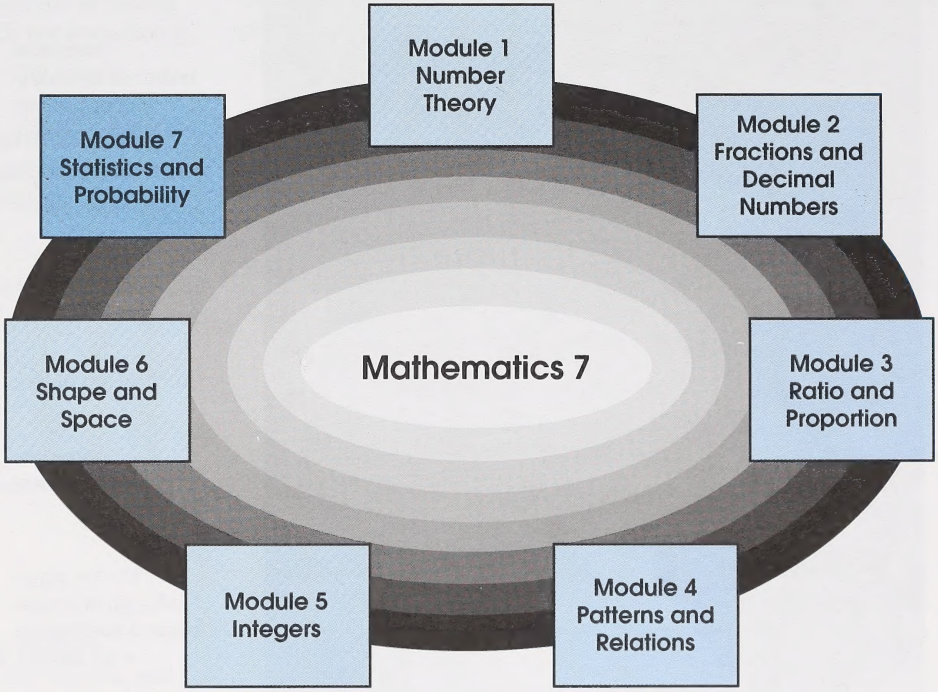
Welcome to Module 7. We hope you'll enjoy your study of Statistics and Probability.

Mathematics 7 contains seven modules. Work through the modules in the order given, since several concepts build on each other as you progress in the course.



The document you are presently reading is called a Student Module Booklet. You may find visual cues or icons throughout it. Read the following explanations to discover what each icon prompts you to do.

- Prepare for a problem that will provide a change of topic.

- Prepare for a challenging problem related to the topic of the activity.


- Use the Internet to explore a topic.

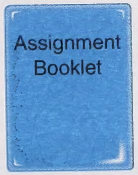
- Use computer software.

- Use a scientific calculator.

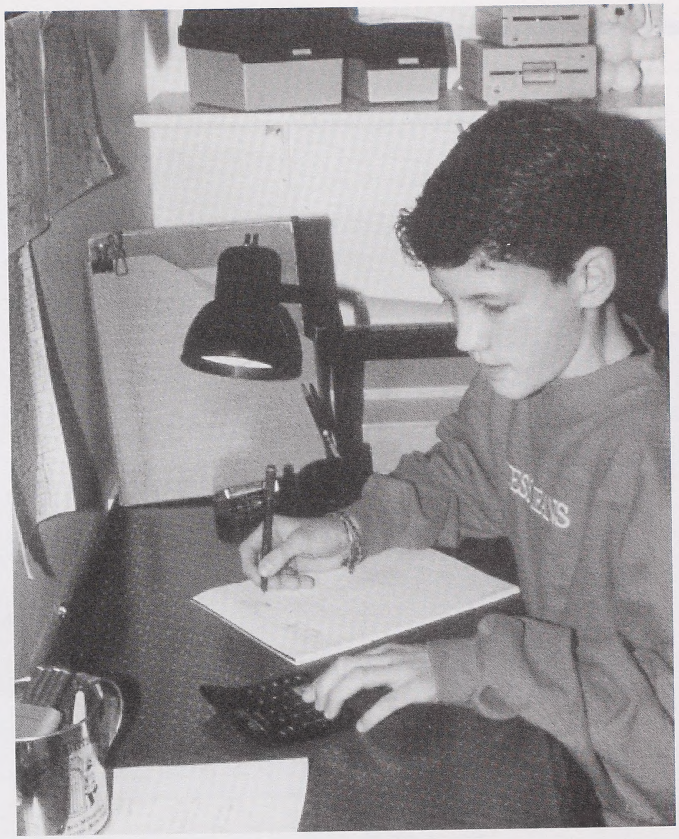
- View a videocassette.

- Pay close attention to important words or ideas.

- Use the suggested answers in the Appendix to correct activities.




- Answer the questions in the Assignment Booklet.



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There are no response spaces provided in this Student Module Booklet. This means that you will need to use your own paper for your responses. You should keep your response pages in a binder so that you can refer to them when you are reviewing or studying.

# Problem-Solving Skills

One of the exciting features of this course is that you will develop and improve your ability in problem solving. You will need these problem-solving skills many times in your lifetime. Since this course focuses on problem solving, it is important that you understand what a **problem** is.

A problem is a task for which the method of finding the answer (as well as the answer) is not immediately known.

Like any skill, the skill of problem solving must be developed. Problems may or may not involve computation (adding, subtracting, multiplying, and dividing). Some problems are realistic; others are puzzles.

You will have the opportunity in most activities to try a problem-solving challenge.

The  icon is a cue that the problem will be related to the topic of the activity.

The  icon is a cue that the problem will provide a change of topic.

# The Four-stage Process

There are four stages that can be used to solve any problem: understanding the problem, developing a plan, trying the plan, and looking back.

## Understanding the Problem

In this stage you should expect to feel puzzled. There are various reasons for feeling this way.

- You may not know the meanings of all the words.
- You may not understand the situation in the problem.
- You may be confused by unnecessary information.

Once you understand the problem, you should think about the problem and make an estimate of what the answer should be. This will help you arrive at a reasonable answer.

## Developing a Plan

This is where you should decide on the plan of action that you are going to take to solve the problem.

You may consider the following strategies:

- changing your point of view
- using objects
- using diagrams
- making an organized list
- using Venn diagrams
- making a table
- guessing, checking, and revising
- acting out a problem
- working backwards
- simplifying a problem
- finding and applying a pattern
- using elimination
- using truth tables
- using an equation

**Note:** The Appendix in Module 1 explains these strategies in detail. When you see a problem-solving icon in any module, you should turn to the Appendix in Module 1 and review the problem-solving strategies.

## Trying the Plan

In this stage you should try the plan and see if it works.

Be sure to work carefully and record your progress. You are encouraged to use a calculator to help with your calculations.

**Note:** While trying the plan, you should monitor your progress in order to determine if your plan will lead to a solution. You may find that the plan will not produce a solution, in which case a new plan will have to be developed.

## Looking Back

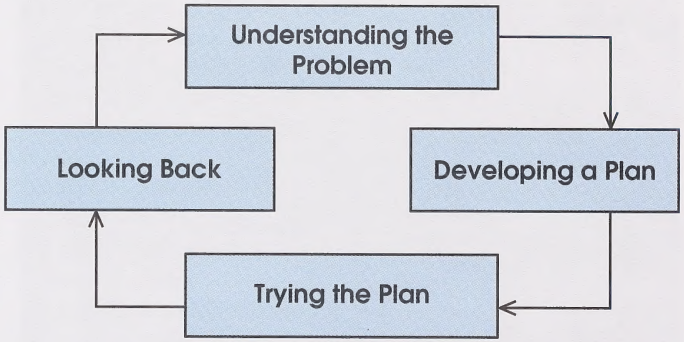
In this stage you should look back at the problem and compare your answer to the estimate you made in the first stage. Restate the problem using your answer.

Ask yourself these questions: "Did my plan work? Is my answer reasonable?"

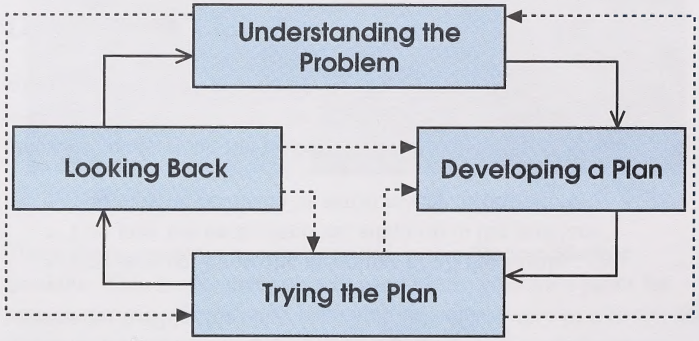
If you did not arrive at an answer, another strategy may work better. If your answer is unreasonable, you may have made errors while trying your plan.

## Sequence of Stages

You usually approach a problem in the order outlined in the following diagram.




If you encounter difficulties in your original plan, or if you realize that another strategy will have better results, you may need to return to an earlier stage or use the stages in a different sequence.



# CONTENTS

# Module Overview

You are living in the information age. Each day you are bombarded with data (pieces of information) as you read, watch television, listen to the radio, or use the computer. Even more information can be found by your own data collection methods.

More than ever before, governmental agencies, businesses, and individuals are concerned with using numerical information to help make decisions. Governments use information on hospital admission levels to decide whether or not a new hospital will be needed in a community in five or ten years or whether the current hospital should be closed. Insurance companies use data on motor vehicle accidents and traffic violations to decide what to charge for insurance. Baseball coaches use team statistics so that they know what pitch to call. You use the weather forecast when planning a day's activities.

The activities in this module will increase your skills in reading and interpreting second-hand data (information collected by others), gathering and reporting first-hand data (information you collect yourself), and making predictions.

## Module 7
## Statistics and Probability

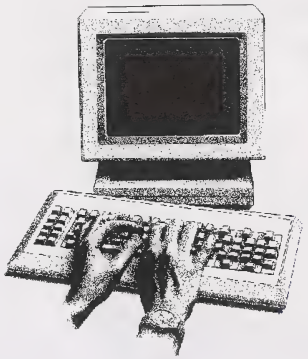
## Evaluation

Your mark for this module will be determined by how well you complete the assignments at the end of each section and at the end of the module. In this module you must complete three assignments. The mark distribution is as follows:

| | |
|---|---|
| **Section 1 Assignment** | **50 marks** |
| **Section 2 Assignment** | **40 marks** |
| **Final Module Assignment** | **10 marks** |
| **TOTAL** | **100 marks** |

When doing the assignments, work slowly and carefully. You must do each assignment independently, but if you are having difficulties, you may review the appropriate section in this module booklet.

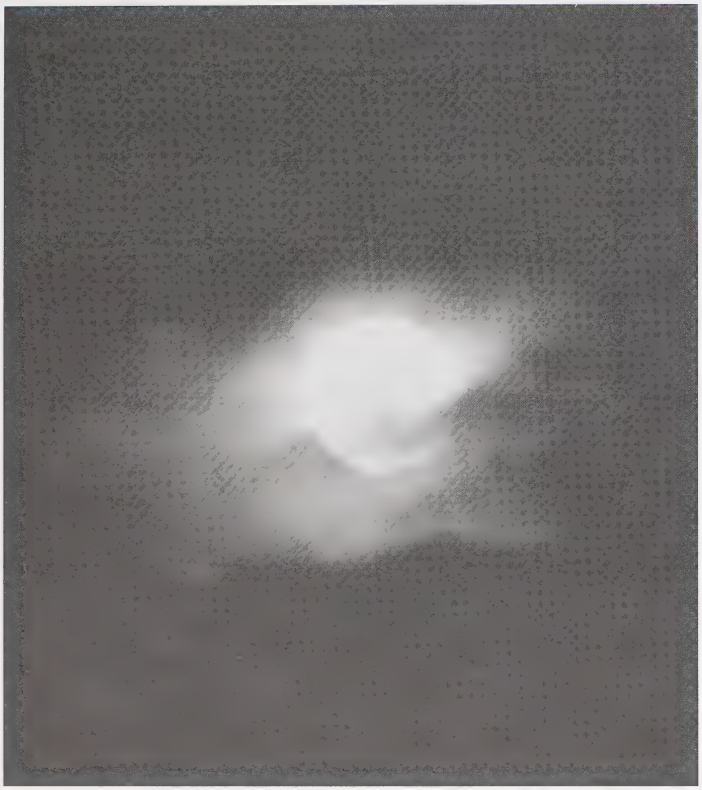
If you are working on a CML terminal, you will have a module test as well as a module assignment.



**Note**

There is a final supervised test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

# Section 1: Second-Hand Data



Earth's moon has fascinated people for thousands of years. The Moon is the closest heavenly body to Earth and, therefore, appears to be the largest in the sky. How far away is the Moon? What is its mass? What is its circumference? What is the temperature on the Moon? How many astronauts have walked on the Moon?

These questions require numerical responses—facts which have been collected by scientists over the years.

In this section, you will use research skills to find the answers to many other questions. You will increase your ability to read tables and graphs. You will also discover new ways to describe sets of data.

# Activity 1: What Is Statistics?

Did you know that the telephone was invented by a Canadian—Alexander Graham Bell?

Here are some **statistics** about telephones.

- By the end of 1991, there were about 537 000 000 telephone subscribers in the world.
- The busiest international telephone route is between the United States and Canada; in 1991 there were 33 000 000 000 minutes of two-way traffic between these two countries.
- Monaco has 810 telephones for every 1000 people. This country has the most telephones per person.

A statistic is an item of information. A number of pieces of information which have been collected and recorded are called statistics (plural of statistic).

The branch of mathematics that deals with the systematic collection and organization of numerical information is also called **statistics**.

**Statistics Canada** is the Canadian agency that collects, analyses, and publishes data about Canadians. The publications include pamphlets, books, compact disks such as *E-STAT*, and articles and **databases** on the Internet.

A computer database is a collection of data organized for rapid search and retrieval. A database may be called a **data bank**.

Statistical information can be found in most libraries. Your library may have the following sources of data:

- encyclopedias and atlases, including multi-media versions
- Statistics Canada publications such as *Canada Year Book*
- newspapers and magazines
- books such as *The Guinness Book of Records*
- almanacs such as *The Canadian Global Almanac*, *The World Almanac and Book of Facts*, or *The Information Please Sports Almanac*
- the Internet

1. Visit a library and find each of the following pieces of information. Tell what source you used to find each piece of information.

   **a.** the greatest distance a baby carriage has been pushed in 24 h (**Hint:** In Britain, a baby carriage is called a pram.)

   **b.** a list of the 10 largest freshwater lakes in the world

   **c.** the Men's World Figure Skating champion in 1995

   **d.** the government expenditures on culture by each province or territory in a recent year

   **e.** the average number of hours per week Canadians spent watching television in a recent year

   **f.** the location of the busiest airport in the world

   **g.** the current population of Ottawa-Hull

   **h.** the motion picture (movie) that earned the most money

2. Explain why two different references may give different answers to questions such as "What is the current population of Ottawa-Hull?" or "What are the ten largest freshwater lakes in the world?"

Check your answers by turning to the Appendix.

## Tables

You will often see data displayed in tables. The data may be listed in alphabetical order, in chronological order (in order of events happening), in geographical order, or in order of size.

Sometimes brief explanations are printed below tables so that the reader is not confused by the data.

Study each of the following tables and answer the questions.

3.

**Age of Mother at Birth of First Child***

| Age of mother | 1931 | 1950 | 1971 | 1990 |
|---|---|---|---|---|
| Under 15 | 14 | 15 | 292 | 233 |
| 15–19 | 9 639 | 14 251 | 33 258 | 19 375 |
| 20–24 | 25 224 | 41 018 | 65 618 | 48 555 |
| 25–29 | 13 826 | 24 330 | 32 918 | 68 647 |
| 30–34 | 4 802 | 8 558 | 7 236 | 30 629 |
| 35–39 | 1 580 | 3 086 | 1 830 | 7 338 |
| 40–44 | 342 | 677 | 380 | 828 |
| 45 and over | 27 | 37 | 15 | 14 |
| Total first-borns** | 55 486 | 92 018 | 142 008 | 175 636 |

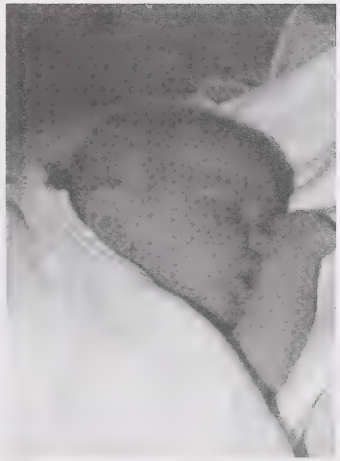
* Excludes Newfoundland
** Includes births for which age of mother is not stated

SOURCE OF DATA: STATISTICS CANADA

**a.** What are the eight age intervals into which the data are divided?

**b.** What age interval had the highest number of first-borns in 1931? in 1950? in 1971? in 1990?

**c.** Why do you think data for Newfoundland were not included?



4.

**Most Popular Baby Names in Canada, 1950–1990**
(in order of popularity)

| 1950 | | 1970 | | 1990 | |
|---|---|---|---|---|---|
| **Boys** | **Girls** | **Boys** | **Girls** | **Boys** | **Girls** |
| Robert | Linda | Michael | Lisa | Michael | Jessica |
| David | Patricia | David | Michelle | Matthew | Amanda |
| John | Barbara | Robert | Jennifer | Christopher | Sarah |
| James | Susan | Jason | Tracy | Andrew | Stephanie |
| William | Sharon | James | Tammy | Kyle | Samantha |
| Richard | Margaret | Christopher | Karen | Ryan | Ashley |
| Kenneth | Donna | John | Nicole | Joshua | Brittany |
| Donald | Judith | Richard | Christine | Daniel | Jennifer |
| Ronald | Carol | Kevin | Shannon | Jordan | Nicole |
| Douglas | Sandra | Mark | Susan | Justin | Kayla |
| Michael | Wendy | Steven | Angel | David | Melissa |
| Brian | Karen | William | Laura | Tyler | Megan |
| Gordon | Shirley | Paul | Sandra | James | Michelle |
| Thomas | Elizabeth | Brian | Tanya | Nicholas | Rebecca |
| Gary | Kathleen | Scott | Heather | Robert | Emily |

**a.** Linda was the most popular name for girls in 1950. Did the name *Linda* rank in the top 15 names in 1970 or in 1990? If so, what was the rank?

**b.** Robert was the most popular name for boys in 1950. Did the name *Robert* rank in the top 15 names in 1970 or 1990? If so, what was the rank?

**c.** What names besides Robert have continued to rank in the top 15 in 1950 to 1990?

5.

**Natural Gas Production* by Province or Territory**

(millions of cubic metres and millions of dollars)

| | 1960 | | 1970 | | 1980 | | 1990 | |
|---|---|---|---|---|---|---|---|---|
| | Quantity | Value | Quantity | Value | Quantity | Value | Quantity | Value |
| Canada | 14 814 | 53 | 52 432 | 292 | 69 820 | 5 737 | 98 771 | 5 692 |
| New Brunswick | 3 | ... | 3 | ... | 2 | ... | — | — |
| Ontario | 481 | 7 | 483 | 6 | 363 | 27 | 449 | 47 |
| Saskatchewan | 1 036 | 4 | 1 522 | 7 | 1 203 | 20 | 5 648 | 306 |
| Alberta | 10 869 | 34 | 42 743 | 250 | 60 517 | 5 370 | 82 214 | 4 842 |
| British Columbia | 2 424 | 8 | 7 679 | 29 | 7 374 | 276 | 10 335 | 491 |
| Northwest Territories | 1 | ... | 2 | ... | 361 | 44 | 124 | 7 |

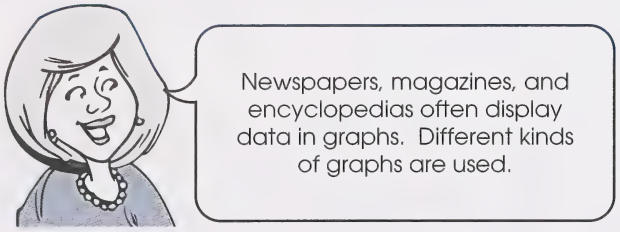
* marketable natural gas
(...) too small to be included
(—) zero

SOURCE OF DATA: STATISTICS CANADA

**a.** How much marketable natural gas was produced in Canada in 1990? What was the value of the gas?
**b.** How much marketable natural gas was produced in New Brunswick in 1990? What was the value of the gas?
**c.** How much marketable natural gas was produced in the Northwest Territories in 1970? What was the value of the gas?
**d.** Which province or territory was the leading producer of natural gas in 1990?
**e.** Which province(s) or territory had a decrease in the production of marketable natural gas from 1980 to 1990?
**f.** Which province(s) or territory had an increase in the production of marketable natural gas from 1980 to 1990?

Check your answers by turning to the Appendix.

## Graphs




Pictographs and bar graphs usually compare the number or size of people or things. The data may be arranged alphabetically, geographically, or in order of rank.

Circle graphs and histograms show how a set of data is distributed. The data in circle graphs are often arranged alphabetically. The data in histograms are usually arranged in order of time or in order of size.

Broken-line graphs usually show how a set of data changes over time.

Use the given pictographs to answer questions 6 and 7.

**6. a.** How are the data in this pictograph arranged?

**b.** Which team won the Stanley Cup the greatest number of times from 1960 to 1990?

**c.** How many times did the Edmonton Oilers win the Stanley Cup in this time period?

**d.** How many times did the Calgary Flames win the Stanley Cup in this time period?

**Stanley Cup Winners (1960–1990)**

| | |
|---|---|
| Boston Bruins | |
| Calgary Flames | |
| Chicago Blackhawks | |
| Edmonton Oilers | |
| Montreal Canadiens | |
| New York Islanders | |
| Philadelphia Flyers | |
| Toronto Maple Leafs | |

Legend: Each represents 1 Stanley Cup.

7.

**Number of Passengers at Canada's Busiest Airports (1985)**

| | |
|---|---|
| Toronto | |
| Vancouver | |
| Montreal (Dorval) | |
| Calgary | |
| Winnipeg | |

Legend: One represents 100 000 people.

**a.** What is being compared in this graph?

**b.** How are the data in this pictograph arranged?

**c.** If represents 100 000 passengers, what do , , and represent?

**d.** About how many passengers used Toronto's airport?

**e.** About how many passengers used Vancouver's airport?

**f.** About how many more passengers used Toronto's airport than Winnipeg's airport?

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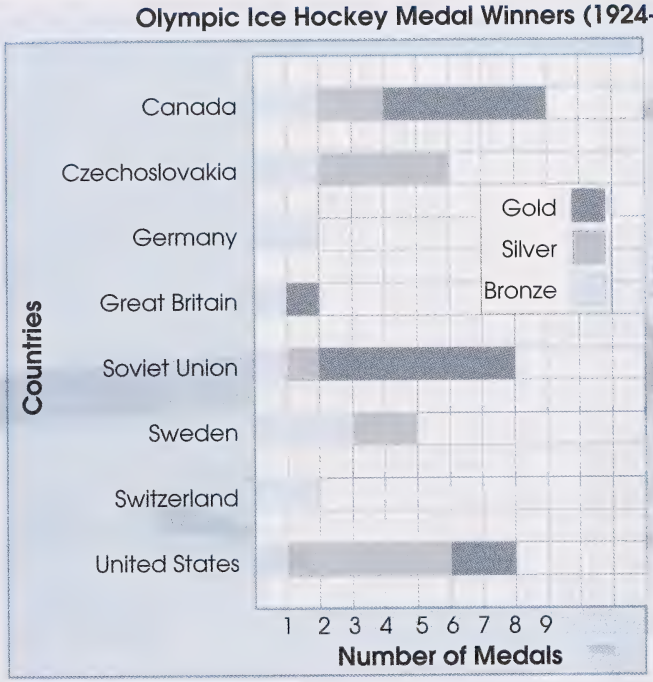
Check your answers by turning to the Appendix.

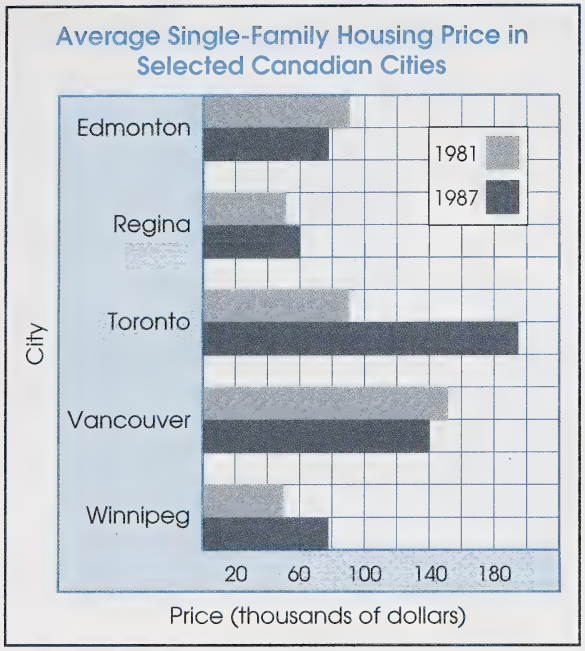
Use the given bar graphs to answer questions 8 to 10.

8.




**a.** What is being compared in this bar graph?
**b.** How are the data in the bar graph arranged?
**c.** Which country won the most ice hockey medals from 1924 to 1984?
**d.** Which countries won gold medals in ice hockey during this period of time?
**e.** Which country won the most gold medals during this period of time?
**f.** How many gold medals did Canada win during this period of time?
**g.** How many silver medals did Canada win during this period of time?
**h.** How many bronze medals did Canada win during this period of time?

**9.**




**a.** What is being compared in the bar graph?

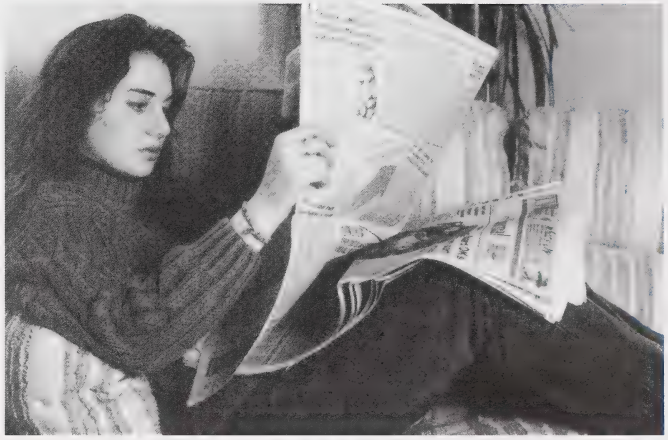
**b.** How are the data in this bar graph arranged?

**c.** In which city did houses cost the most in 1981? in 1987?


**d.** Where did the house prices increase in this time period? Which city showed the largest increase?

**e.** Where did house prices decrease in this time period? Which city showed the largest decrease?

**Note:** Many city newspapers publish weekly housing statistics. Examine a city newspaper to discover what types of housing statistics are published.



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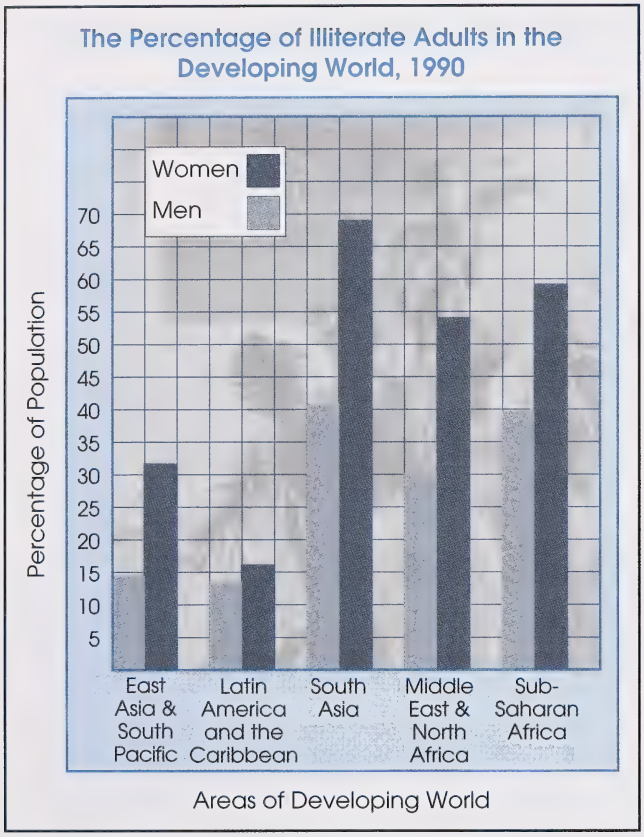
Use the Internet to discover data about the housing market in Canada. **Hint:** This is the uniform resource locator (URL) of a site which you may find interesting. It was developed by the Canadian Mortgage and Housing Corporation.

http://www.cmhc-schl.gc.ca/cmhc.html

**10.**




SOURCE OF DATA: CIDA

**a.** What is being compared in this bar graph?

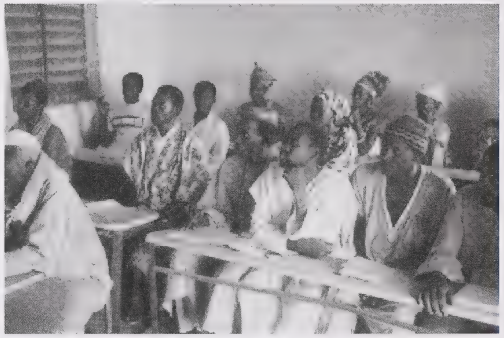
**b.** In 1990, which area of the developing world had the greatest percentage of illiteracy for men? for women?

**c.** In 1990, which area of the developing world had the least percentage of illiteracy for men? for women?

**d.** According to the graph, how did illiteracy for men and women compare in these areas of the developing world?

**e.** In 1990, which area of the developing world had the greatest difference in the percentage of men and women who were illiterate?

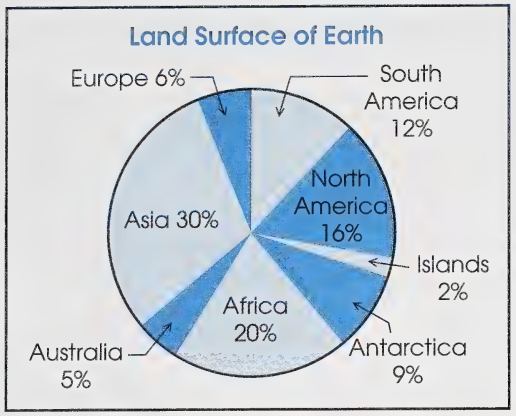
**f.** In 1990, which area of the developing world had the least difference in the percentage of men and women who were illiterate?



Check your answers by turning to the Appendix.

Use the given circle graph and histograms to answer questions 11 to 13.

**11.**



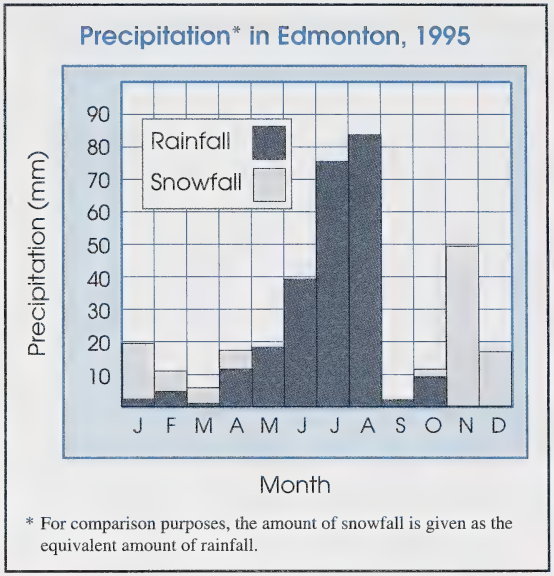

**a.** What is being shown in this circle graph?

**b.** Which continent has the greatest amount of land surface?

**c.** Where does North America rank in respect to the amount of land surface?

**d.** If there are about 155 400 000 $km^2$ of land surface on Earth, find the number of square kilometres of land surface in North America.

**12.**




**a.** What is being compared in this histogram?

**b.** Which month had the most precipitation?

**c.** How much precipitation was there in Edmonton in April, 1995? Of this precipitation, how much was rainfall? How much was snowfall (in rainfall equivalent)?

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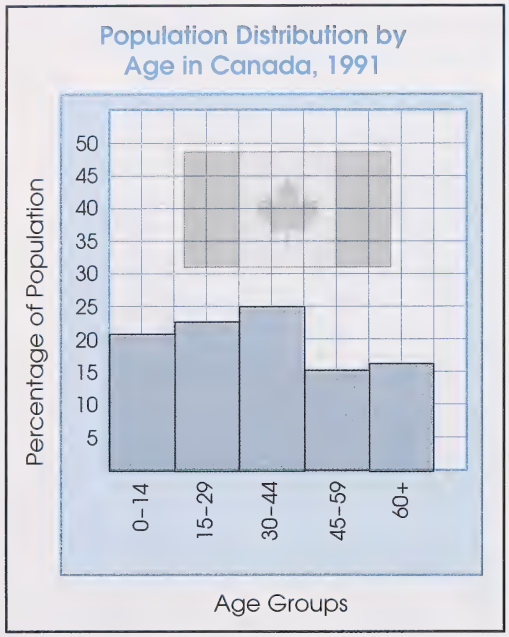
**13.**




**a.** What is being shown in this histogram?

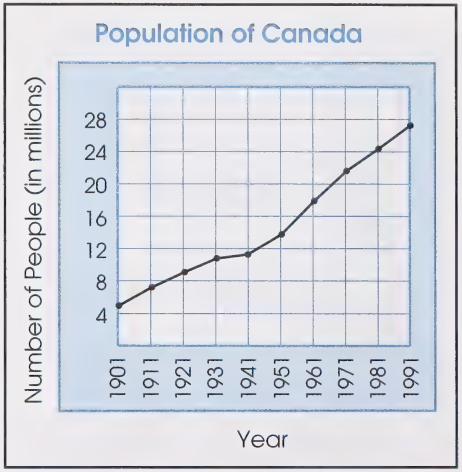
**b.** How are the data arranged?

**c.** Order the age groups from the greatest percentage of the population to the least.

Check your answers by turning to the Appendix.

Use the given broken-line graphs to answer questions 14 and 15.

**14.**




SOURCE OF DATA: STATISTICS CANADA

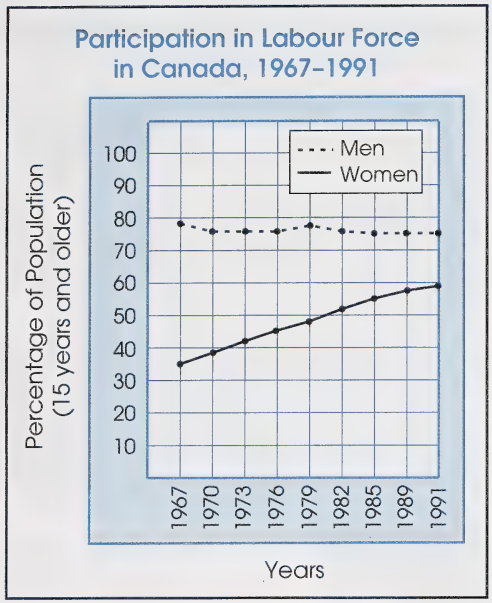
**a.** What is being shown in this broken-line graph?

**b.** How are the data arranged?

**c.** Estimate the population of Canada in 1911, 1931, 1951, 1971, and 1991.

**d.** What trend does the graph show? (That is, is the population of Canada decreasing, remaining the same, or increasing?)

**15.**




SOURCE OF DATA: STATISTICS CANADA

**a.** What percentage of men (15 years and over) were part of the labour force in 1967? in 1991?

**b.** Did the percentage of men (15 years and over) who participated in the labour force increase, decrease, or remain about the same from 1967 to 1991?

**c.** Predict the percentage of men (15 years and over) who will be participating in the labour force in 30 years time.

**d.** What percentage of women (15 years and over) were part of the labour force in 1967? in 1991?

**e.** Did the percentage of women (15 years and over) who participated in the labour force increase, decrease, or remain about the same from 1967 to 1991?

**f.** Predict the percentage of women (15 years and over) who will be participating in the labour force in 30 years time.

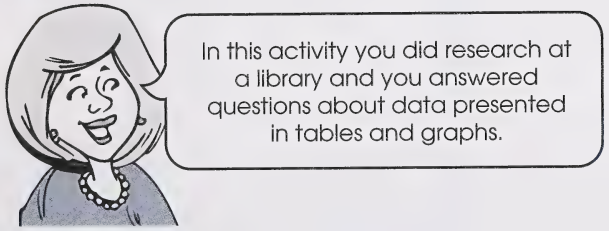
Check your answers by turning to the Appendix.

## Now Try This

Use a problem-solving strategy to answer the following question.

**16.** Four friends went camping. Ashley set up his tent 25 m due west of a flagpole. Brenda set up her tent 25 m due north of Ashley. Chelsea set up her tent 50 m due east of Brenda. Dan set up his tent 25 m due south of Chelsea. How far was Dan from the flagpole?

Check your answer by turning to the Appendix.




## Activity 2: Finding the Average of a Set of Data

Did you know that the African elephant is presently the largest living land animal? An African elephant can weigh up to 7500 kg and may stand as high as 4 m at the shoulder. Records such as the heaviest elephant or the tallest elephant are interesting, but often it is more helpful to know the average mass or height.

In this activity you will investigate three kinds of averages.

### Mean

The most commonly used average is the **mean**.

The mean is the arithmetic average.

## Example 1

Jean is a soccer goalie. Following is a table of the number of goals scored against him in 7 games.

What was the mean number of goals scored against Jean in these games?

| Soccer Results | | | | | | | |
|---|---|---|---|---|---|---|---|
| Game | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Goals Scored | 3 | 5 | 2 | 5 | 0 | 2 | 4 |

## Solution

**Step 1:** Find the total number of goals scored in 7 games.

$$3+5+2+5+0+2+4=21$$

Jean allowed 21 goals in 7 games.

**Step 2:** Find the mean by dividing the total number of goals by the number of games.

$$\frac{21}{7}=3$$

The mean number of goals that Jean allowed was 3.

Use the table in Example 1 to answer question 1.

**1. a.** In how many games did Jean allow 3 goals?

**b.** In how many games did Jean allow fewer than 3 goals?

**c.** In how many games did Jean allow more than 3 goals?

**2.** Sally likes to bowl. This is a table of her scores in 5 games of bowling.

| Bowling Results | | | | | |
|---|---|---|---|---|---|
| Game | 1 | 2 | 3 | 4 | 5 |
| Score | 80 | 96 | 84 | 92 | 90 |

**a.** Give the mean of Sally's scores.

**b.** In how many games did Sally make this mean score?

**c.** In how many games did Sally make more than this mean score?

**d.** In how many games did Sally make less than this mean score?

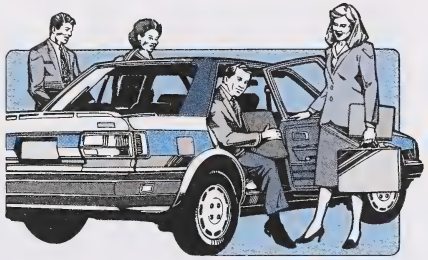
Check your answers by turning to the Appendix.

Sometimes it is helpful to make a line plot in order to find the mean.

A line plot is a simple way of organizing and displaying a set of data.

## Example 2

Eugene counted the number of people in each of the 35 vehicles that entered the parking lot at his office at the beginning of a working day. He made a list of these numbers.

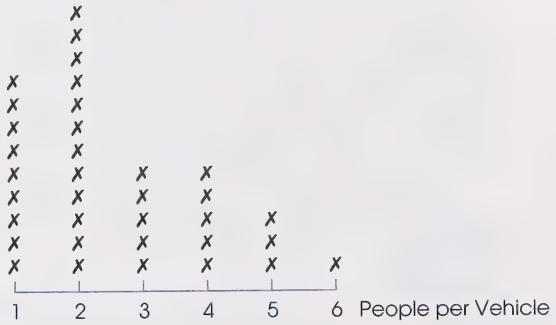


| | | | | | | |
|---|---|---|---|---|---|---|
| 2 | 3 | 4 | 2 | 3 | 2 | 2 |
| 2 | 2 | 1 | 3 | 2 | 2 | 4 |
| 2 | 1 | 5 | 2 | 3 | 6 | 1 |
| 4 | 2 | 1 | 4 | 1 | 1 | 5 |
| 1 | 3 | 5 | 1 | 2 | 4 | 1 |

Give the mean number of people per vehicle.

## Solution

**Step 1:** Record the data in a line plot like this. **Hint:** One at a time, record the numbers in the list. The first number is 2, so put an ✗ above 2 on the line plot. The next number is 3, so put an ✗ above 3 on the line plot. Continue this process until all the numbers have been recorded.




**Step 2:** Use the line plot to find the total number of people in all the vehicles.

$$(9\times1)+(12\times2)+(5\times3)+(5\times4)+(3\times5)+6$$
$$=9+24+15+20+15+6$$
$$=89$$

**Step 3:** Find the mean. Round to the nearest whole number.

$$\frac{89}{35}\doteq 3$$

Divide the total number of people by the number of vehicles.

The mean number of people per vehicle was about 3.

**3.** The 25 students in Grade 7C were asked how many fillings they had ever had in their teeth. Their responses are shown in the following list.

| | | | | |
|---|---|---|---|---|
| 3 | 5 | 8 | 2 | 0 |
| 6 | 7 | 4 | 0 | 8 |
| 6 | 1 | 1 | 4 | 1 |
| 3 | 0 | 7 | 1 | 1 |
| 2 | 3 | 0 | 0 | 0 |

Give the mean number of fillings per student.

Check your answers by turning to the Appendix.

## Median

Another commonly used average is the **median**.

To find the median, arrange the values in order from least to greatest. If there is an odd number of values, the median is the middle value. If there is an even number of values, the median is the mean of the two middle values.

## Example 1

In a gym class, 5 students took five free throws each at the basket on a basketball court. The number of successful free throws per student is given in the following table.

Give the median number of successful free throws per student.

| Student | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Number of successful free throws | 3 | 4 | 1 | 1 | 2 |

### Solution

**Step 1:** Arrange the list of the number of successful free throws in order from least to greatest.

1 1 2 3 4

**Step 2:** There is an odd number of values in the data. So, circle the middle value.

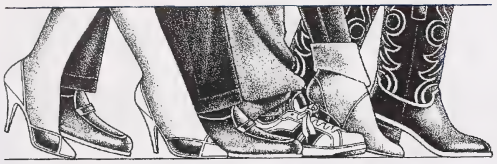
1 1 (2) 3 4

Median

There are two values less than the median and two values greater than the median.

The median number of successful free throws per student was 2.

## Example 2



In one hour a shoe store sold 8 pairs of shoes. Following is a list of the shoe sizes sold.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 8 | 9 | 8 | 7 | 9 | 11 | 10 | 8 |

Give the median shoe size sold.

### Solution

**Step 1:** Arrange the list of shoe sizes in order from least to greatest.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 8 | 8 | 8 | 9 | 9 | 10 | 11 |

**Step 2:** There is an even number of values in the data, so find the median by putting a line between the middle two values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 8 | 8 | 8 | \| | 9 | 9 | 10 | 11 |

There are four values less than the median and four values greater than the median. The median is between 8 and 9.

**Step 3:** Find the mean of the middle two values.

$$\frac{8+9}{2} = \frac{17}{2}$$
$$= 8.5$$

The median shoe size is 8.5.

**4.** 

Following is a list of the number of petals on each of 9 daisies. Give the median number of petals per daisy.

| | | | | |
|---|---|---|---|---|
| 18 | 19 | 16 | 18 | 19 |
| 17 | 16 | 19 | 18 | |

**5.** Following is a list of the number of attempts it took each of 10 people in a dart competition to hit the bull's-eye. Give the median number of attempts per person before scoring a bull's-eye.

| | | | | |
|---|---|---|---|---|
| 6 | 6 | 8 | 3 | 4 |
| 3 | 1 | 7 | 2 | 5 |

Check your answers by turning to the Appendix.

Sometimes it is helpful to make a line plot in order to find the median.

## Example 3

Following is a list of the number of kittens born in each of 30 litters.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 4 | 3 | 5 | 3 | 3 | 3 | 3 | 4 |
| 4 | 3 | 5 | 4 | 4 | 2 | 3 | 1 | 3 | 2 |
| 3 | 3 | 2 | 2 | 5 | 2 | 4 | 2 | 2 | 3 |

Give the median number of kittens per litter.

## Solution

**Step 1:** Record the data in a line plot like this.


**Step 2:** Use the line plot to arrange the number of kittens per litter in order from least to greatest.

There are 2 litters of 1 kitten, 7 litters of 2 kittens, 12 litters of 3 kittens, 6 litters of 4 kittens, and 3 litters of 5 kittens.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 3 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 |

**Step 3:** Because there is an even number of values in the data, put a line between the middle two values. Then find the mean of the middle two values.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 2 | 2 | 2 | 2 | 2 | 2 | 3 |
| 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 |
| 3 | 4 | 4 | 4 | 4 | 4 | 4 | 5 | 5 | 5 |

The median is between 3 and 3.

$$\frac{3+3}{2} = \frac{6}{2}$$
$$= 3$$

The median number of kittens per litter was 3.

**6.** Following is a list of the high temperatures in a certain city during July, 1995.

| | | | | | |
|---|---|---|---|---|---|
| 22 | 29 | 24 | 22 | 25 | 20 |
| 28 | 24 | 27 | 21 | 23 | 25 |
| 20 | 27 | 26 | 26 | 27 | 23 |
| 20 | 23 | 26 | 27 | 28 | 24 |
| 26 | 20 | 21 | 23 | 25 | 22 |
| 22 | | | | | |

Give the median high temperature for this time period.

Check your answers by turning to the Appendix.

Sometimes it is helpful to make a **stem and leaf plot** in order to find the median.

A stem and leaf plot is a way of organizing and presenting a set of data in a T-table.

**Note:** When you make a stem and leaf plot, you may organize the values in intervals of 5 or intervals of 10.

## Example 4

Each year the American Academy of Motion Picture Arts and Sciences presents Oscars. Following is a list of the ages of women who won the best actress award for the years 1970 to 1994.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 34 | 34 | 26 | 37 | 42 | 41 | 35 | 31 | 41 |
| 33 | 30 | 74 | 33 | 49 | 38 | 61 | 21 | 41 |
| 26 | 80 | 42 | 29 | 33 | 36 | 46 | | |

Give the median age of the women winning an Oscar in the best actress category for the years 1970 to 1994.

## Solution

**Step 1:** Record the data in a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 2 | 1 |
| • | 6, 6, 9 |
| 3 | 4, 4, 1, 3, 0, 3, 3 |
| • | 7, 5, 8, 6 |
| 4 | 2, 1, 1, 1, 2 |
| • | 9, 6 |
| 5 | |
| • | |
| 6 | 1 |
| • | |
| 7 | 4 |
| • | |
| 8 | 0 |

This stem and leaf plot has intervals of 5.

**Step 2:** Order the numbers in each leaf of the stem and leaf plot from least to greatest.

| Stem | Leaf |
|---|---|
| 2 | 1 ← This leaf is read as "21." |
| • | 6, 6, 9 ← This leaf is read as "26, 26, 29." |
| 3 | 0, 1, 3, 3, 3, 4, 4 ← This leaf is read as "30, 31, 33, 33, 33, 34, 34." |
| • | 5, 6, 7, 8 |
| 4 | 1, 1, 1, 2, 2 |
| • | 6, 9 |
| 5 | |
| • | |
| 6 | 1 |
| • | |
| 7 | 4 |
| • | |
| 8 | 0 |

**Step 3:** Use the stem and leaf plot to arrange the ages in order from least to greatest.

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 26 | 26 | 29 | 30 | 31 | 33 |
| 33 | 33 | 34 | 34 | 35 | 36 | 37 |
| 38 | 41 | 41 | 41 | 42 | 42 | 46 |
| 49 | 61 | 74 | 80 | | | |

**Step 4:** There is an odd number of values. So, circle the middle value.

| | | | | | | |
|---|---|---|---|---|---|---|
| 21 | 26 | 26 | 29 | 30 | 31 | 33 |
| 33 | 33 | 34 | 34 | 35 | (36) | 37 |
| 38 | 41 | 41 | 41 | 42 | 42 | 46 |
| 49 | 61 | 74 | 80 | | | |

The median age was 36 years.

## Example 5

Following is a list of the scores of 24 golfers after an 18-hole tournament.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 84 | 79 | 74 | 85 | 97 | 67 | 92 | 96 |
| 69 | 100 | 88 | 93 | 87 | 104 | 94 | 106 |
| 94 | 67 | 78 | 98 | 79 | 102 | 72 | 71 |

Give the median score of these golfers.

## Solution

**Step 1:** Record the data in a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 6 | 7, 9, 7 |
| 7 | 9, 4, 8, 9, 2, 1 |
| 8 | 4, 5, 8, 7 |
| 9 | 7, 2, 6, 3, 4, 4, 8 |
| 10 | 0, 4, 6, 2 |

This stem and leaf plot has intervals of 10.

**Step 2:** Order the numbers in each leaf of the stem and leaf plot from least to greatest.

| Stem | Leaf | |
|---|---|---|
| 6 | 7, 7, 9 | ← This leaf is read as "67, 67, 69." |
| 7 | 1, 2, 4, 8, 9, 9 | ← This leaf is read as "71, 72, 74,... ." |
| 8 | 4, 5, 7, 8 | |
| 9 | 2, 3, 4, 4, 6, 7, 8 | |
| 10 | 0, 2, 4, 6 | |

**Step 3:** Use the stem and leaf plot to arrange the scores in order from least to greatest.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 67 | 67 | 69 | 71 | 72 | 74 | 78 | 79 |
| 79 | 84 | 85 | 87 | 88 | 92 | 93 | 94 |
| 94 | 96 | 97 | 98 | 100 | 102 | 104 | 106 |

**Step 4:** There is an even number of values in the data. So, find the median by putting a line between the middle two values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 67 | 67 | 69 | 71 | 72 | 74 | 78 | 79 |
| 79 | 84 | 85 | 87 | 88 | 92 | 93 | 94 |
| 94 | 96 | 97 | 98 | 100 | 102 | 104 | 106 |

There are twelve values less than the median and twelve values greater than the median.

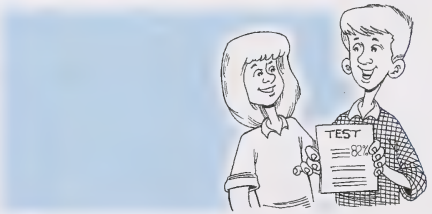
The median score was between 87 and 88.

**Step 5:** Find the mean of the middle two values.

$$\frac{87+88}{2} = \frac{175}{2}$$
$$= 87.5$$

The median score was 87.5.

**7.** Following are the scores made by 30 students on an English assignment.




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 48 | 38 | 43 | 68 | 64 | 68 | 51 | 68 | 54 | 53 |
| 53 | 59 | 80 | 82 | 82 | 77 | 54 | 79 | 89 | 72 |
| 57 | 77 | 82 | 73 | 79 | 86 | 81 | 76 | 85 | 56 |

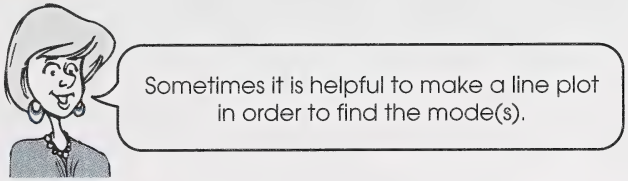
Give the median score on the English assignment.

Check your answer by turning to the Appendix.

## Mode

So far you have investigated the mean and the median. Another kind of average is the **mode**.

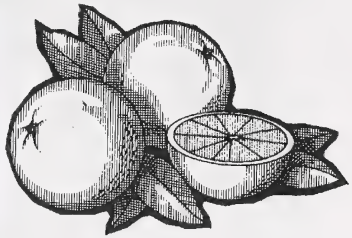
The mode is the most frequently occurring value or values in a set of data. When a set of values has two modes, it is called a **bimodal distribution**. When a set of values has three modes, it is called a **trimodal distribution**.




## Example

Following is a list of the number of seeds in each of 30 oranges.

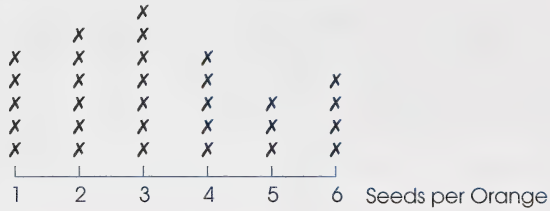
| | | | | |
|---|---|---|---|---|
| 4 | 2 | 6 | 2 | 3 |
| 3 | 3 | 4 | 2 | 3 |
| 5 | 6 | 1 | 3 | 4 |
| 6 | 1 | 5 | 4 | 2 |
| 6 | 3 | 2 | 5 | 3 |
| 1 | 1 | 4 | 2 | 1 |



Give the mode(s) number of seeds per orange.

## Solution

**Step 1:** Record the data in a line plot.




**Step 2:** Use the line plot to find which number of seeds occurs most frequently. **Hint:** Which column has the most ✗s?

The mode is 3 seeds per orange.

Use a line plot to answer each of the following questions.

**8.** Following is a list of the number of students in each of the 20 classes at Douglas Street School.

| | | | | |
|---|---|---|---|---|
| 25 | 27 | 28 | 30 | 25 |
| 26 | 27 | 30 | 29 | 27 |
| 26 | 30 | 27 | 28 | 29 |
| 30 | 27 | 25 | 30 | 29 |

Give the mode(s) of the number of students in a class.

**9.** Leon volunteers for his community's Meals on Wheels program.

The following list shows the number of hours he worked each day for the past 20 days.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 3 | 3 | 3 | 7 | 0 | 3 | 3 | 3 |
| 5 | 5 | 6 | 0 | 3 | 3 | 3 | 4 | 2 | 5 |

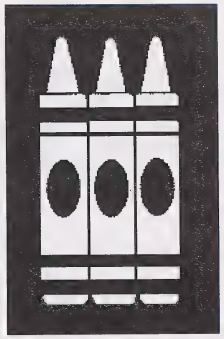
Give the mode(s) of the number of hours Leon worked.

Check your answers by turning to the Appendix.

## Now Try This

Use a problem-solving strategy to answer the following question.

**10.** Three colours of markers were chosen—red, blue, and green. Art, Bill, and Clive each chose a different colour. Art and the boy who chose blue are the same age. No person's name has the same number of letters as the number of letters in the colour he chose. What colour of marker did each boy choose?



Check your answer by turning to the Appendix.

Often it is useful to find an average for a set of data. In this activity you found three different kinds of averages—the mean, the median, and the mode. These averages are sometimes called **measures of central tendency**.

# Activity 3: Describing the Distribution of a Set of Data

Humans vary in size and shape. Some are tall; others are short. The tallest adult whose height has been officially recorded was Robert Wadlow (1918–1940). When he was last measured, he was 272 cm tall. The shortest adult on record is Gul Mohammed (1957– ). When he was last measured, he was 57 cm tall.


You may discover more about Robert Wadlow and view some interesting photos of him by visiting this site:

http://www.altonweb.com/visitors/wadlow.htm

In statistics, the least value in a set of data is the **lower extreme**. The **upper extreme** is the greatest value in the data. The difference between the upper extreme and the lower extreme is the **range**.

The lower extreme of human adult heights is 57 cm and the upper extreme is 272 cm.

$$272 - 57 = 215$$

So, the range of human heights is 215 cm.

**1.** Use this table to answer the given questions.

**Entry Dates of the Provinces and Territories into Confederation**

| Province or Territory | Date |
|---|---|
| Alberta | 1905 |
| British Columbia | 1871 |
| Manitoba | 1870 |
| New Brunswick | 1867 |
| Newfoundland | 1949 |
| Northwest Territories | 1870 |
| Nova Scotia | 1867 |
| Ontario | 1867 |
| Prince Edward Island | 1873 |
| Quebec | 1867 |
| Saskatchewan | 1905 |
| Yukon | 1898 |

**a.** Give the lower extreme of the entry dates.

**b.** Give the upper extreme of the entry dates.

**c.** Give the range of the entry dates of the provinces and territories.

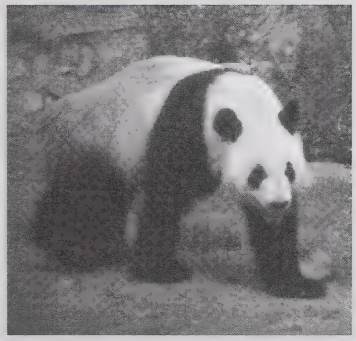

**2.** Following is a list of the number of visitors at the Calgary Zoo during June, July, August, and September 1988 when the Giant Pandas were on exhibit.

162 184 268 726 296 330 71 940 [1]

**a.** Give the lower extreme of the number of visitors per month.

**b.** Give the upper extreme of the number of visitors per month.

**c.** Give the range of the number of visitors.



[1] Calgary Zoo for the data

**3.** Use this table to answer the following questions.

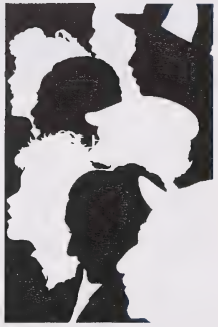
| Populations, 1991 | |
|---|---|
| **Province or Territory** | **Population** |
| Newfoundland | 568 000 |
| Prince Edward Island | 130 000 |
| Nova Scotia | 900 000 |
| New Brunswick | 924 000 |
| Quebec | 6 896 000 |
| Ontario | 10 085 000 |
| Manitoba | 1 092 000 |
| Saskatchewan | 989 000 |
| Alberta | 2 546 000 |
| British Columbia | 3 282 000 |
| Yukon | 28 000 |
| Northwest Territories | 58 000 |

SOURCE OF DATA: STATISTICS CANADA

**a.** Give the lower extreme of the populations.

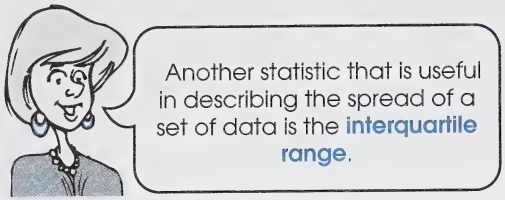
**b.** Give the upper extreme of the populations.

**c.** Give the range of the populations.



Check your answers by turning to the Appendix.

The lower extreme, upper extreme, and the range give information about how spread out a set of data is.




To find the interquartile range of a set of data, you must first find the median of the values, and then the **lower quartile** and **upper quartile**.

The lower quartile is the median of the values less than the median in the set of data. The upper quartile is the median of the values greater than the median. The interquartile range is the difference between the upper quartile and the lower quartile.

## Example 1

Following is a list of the number of grams of carbohydrates per serving in each of 13 different brands of breakfast cereal.

| | | | | | | |
|---|---|---|---|---|---|---|
| 19 | 26 | 25 | 24 | 23 | 26 | |
| 23 | 21 | 28 | 23 | 13 | 25 | 20 |

Give the median, the lower quartile, the upper quartile, and the interquartile range of this set of data.

## Solution

**Step 1:** Arrange the data in order from least to greatest.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 19 | 20 | 21 | 23 | 23 | |
| 23 | 24 | 25 | 25 | 26 | 26 | 28 |

**Step 2:** Find the median.

There is an odd number of values. So, circle the middle value.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 19 | 20 | 21 | 23 | 23 | |
| (23) | 24 | 25 | 25 | 26 | 26 | 28 |

The median number of grams of carbohydrates per serving is 23 grams.

**Step 3:** Find the lower quartile.

Consider only the data **less** than the median. There is an even number of values less than 23. So, put a line between the middle two values.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 19 | 20 \| | 21 | 23 | 23 | |
| (23) | 24 | 25 | 25 | 26 | 26 | 28 |

The lower quartile is between 20 and 21. Find the mean of these middle two values.

$$\frac{20+21}{2} = \frac{41}{2}$$
$$= 20.5$$

The lower quartile is 20.5 grams of carbohydrates per serving.

**Step 4:** Find the upper quartile.

Consider only the data **greater** than the median. There is an even number of values greater than 23. So, put a line between the middle two values.

| | | | | | | |
|---|---|---|---|---|---|---|
| 13 | 19 | 20 | 21 | 23 | 23 | |
| (23) | 24 | 25 | 25 | 26 | 26 | 28 |

The upper quartile is between 25 and 26. Find the mean of these middle two values.

$$\frac{25+26}{2} = \frac{51}{2}$$
$$= 25.5$$

The upper quartile is 25.5 grams of carbohydrates per serving.

**Step 5:** Find the interquartile range.

Subtract the lower quartile from the upper quartile.

$$25.5 - 20.5 = 5$$

The interquartile range for the set of data is 5 grams of carbohydrates per serving.

**4.** This is a list of the number of goals the Elks scored in each of 14 field hockey games.

0 3 1 4 3 5 4 3 2 3 6 1 4 5

**a.** Give the median of this set of data.

**b.** Give the lower quartile of this set of data.

**c.** Give the upper quartile of this set of data.

**d.** Give the interquartile range of this set of data.

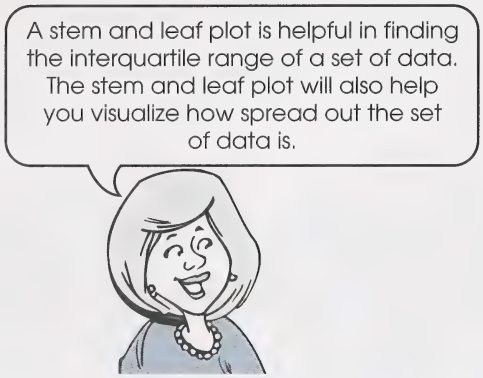
**5.** Following is a list of the number of satellites (moons) per planet. Use this list to answer the given questions.

| | | | | | |
|---|---|---|---|---|---|
| Mercury .... | 0 | Venus ..... | 0 | Earth ..... | 1 |
| Mars ..... | 2 | Jupiter ..... | 16* | Saturn ..... | 18* |
| Uranus ..... | 15 | Neptune ..... | 8 | Pluto ..... | 1 |

*Sources disagree about the number of satellites (moons). This was the most common number given in 1995.

**a.** Give the median number of satellites (moons) per planet.
**b.** Give the lower quartile of this set of data.
**c.** Give the upper quartile of this set of data.
**d.** Give the interquartile range of the data.

Check your answers by turning to the Appendix.




## Example 2

This is a list of the ages of patients in one extended care hospital.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 80 | 85 | 74 | 86 | 86 | 90 | 83 | 60 | 36 | 73 |
| 80 | 96 | 86 | 87 | 73 | 83 | 59 | 85 | 91 | 79 |
| 86 | 70 | 80 | 74 | 92 | 93 | 76 | 77 | 78 | 70 |
| 82 | 85 | 61 | 76 | 90 | 92 | 84 | 86 | 77 | 93 |

Give the range and interquartile range of this set of data.

## Solution

**Step 1:** Record the data in a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 3 | |
| • | 6 |
| 4 | |
| • | |
| 5 | |
| • | 9 |
| 6 | 0, 1 |
| • | |
| 7 | 4, 3, 3, 0, 4, 0 |
| • | 9, 6, 7, 8, 6, 7 |
| 8 | 0, 3, 0, 3, 0, 2, 4 |
| • | 5, 6, 6, 6, 7, 5, 6, 5, 6 |
| 9 | 0, 1, 2, 3, 0, 2, 3 |
| • | 6 |

**Step 2:** Arrange the numbers in each leaf in ascending order.

| Stem | Leaf |
|---|---|
| 3 | |
| • | 6 |
| 4 | |
| • | |
| 5 | |
| • | 9 |
| 6 | 0, 1 |
| • | |
| 7 | 0, 0, 3, 3, 4, 4 |
| • | 6, 6, 7, 7, 8, 9 |
| 8 | 0, 0, 0, 2, 3, 3, 4 |
| • | 5, 5, 5, 6, 6, 6, 6, 6, 7 |
| 9 | 0, 0, 1, 2, 2, 3, 3 |
| • | 6 |

**Step 3:** Use the stem and leaf plot in Step 2 to arrange the values in order from least to greatest.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36 | 59 | 60 | 61 | 70 | 70 | 73 | 73 | 74 | 74 |
| 76 | 76 | 77 | 77 | 78 | 79 | 80 | 80 | 80 | 82 |
| 83 | 83 | 84 | 85 | 85 | 85 | 86 | 86 | 86 | 86 |
| 86 | 87 | 90 | 90 | 91 | 92 | 92 | 93 | 93 | 96 |

**Step 4:** Find the range.

$96 - 36 = 60$

Subtract the lower extreme from the upper extreme.

The range is 60 years.

**Step 5:** Find the median. **Hint:** There is an even number of values in the data. So, put a line between the middle two values. Then find the mean of these two values.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36 | 59 | 60 | 61 | 70 | 70 | 73 | 73 | 74 | 74 |
| 76 | 76 | 77 | 77 | 78 | 79 | 80 | 80 | 80 | 82 \| |
| 83 | 83 | 84 | 85 | 85 | 85 | 86 | 86 | 86 | 86 |
| 86 | 87 | 90 | 90 | 91 | 92 | 92 | 93 | 93 | 96 |

$$\frac{82+83}{2} = \frac{165}{2}$$
$$= 82.5$$

The median is 82.5 years.

**Step 6:** Find the lower quartile. **Hint:** Consider only the values **less** than the median. Because there is an even number of values, put a line between the middle two values. Then find the mean of these two values.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36 | 59 | 60 | 61 | 70 | 70 | 73 | 73 | 74 | 74 \| |
| 76 | 76 | 77 | 77 | 78 | 79 | 80 | 80 | 80 | 82 \| |
| 83 | 83 | 84 | 85 | 85 | 85 | 86 | 86 | 86 | 86 |
| 86 | 87 | 90 | 90 | 91 | 92 | 92 | 93 | 93 | 96 |

$$\frac{74+76}{2} = \frac{150}{2}$$
$$= 75$$

The lower quartile is 75 years.

**Step 7:** Find the upper quartile. **Hint:** Consider only the data **greater** than the median. There is an even number of values. So, put a line between the middle two values. Then find the mean of these two values.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36 | 59 | 60 | 61 | 70 | 70 | 73 | 73 | 74 | 74 \| |
| 76 | 76 | 77 | 77 | 78 | 79 | 80 | 80 | 80 | 82 \| |
| 83 | 83 | 84 | 85 | 85 | 85 | 86 | 86 | 86 | 86 \| |
| 86 | 87 | 90 | 90 | 91 | 92 | 92 | 93 | 93 | 96 |

$$\frac{86+86}{2} = \frac{172}{2}$$
$$= 86$$

The upper quartile is 86 years.

**Step 8:** Find the interquartile range.

$$86 - 75 = 11$$

Subtract the lower quartile from the upper quartile.

The interquartile range is 11 years.

The stem and leaf plot in Example 2 helps you visualize how spread out the set of data is. There is a range of 60 years and an interquartile range of 11 years.

Use stem and leaf plots to answer questions 6 and 7.

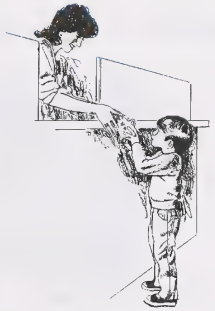
**6.** This is a list of the test scores in a science class.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 83 | 81 | 68 | 82 | 61 | 84 | 62 | 83 |
| 58 | 63 | 96 | 68 | 73 | 80 | 77 | 95 |
| 71 | 38 | 72 | 72 | 78 | 83 | 78 | 63 |
| 60 | 79 | 69 | 74 | 99 | | | |

Give each of the following statistics:

**a.** interquartile range
**b.** range

**7.** Following is a list of amounts of money (rounded to the nearest dollar) in a set of 30 savings accounts owned by children under eight years of age.

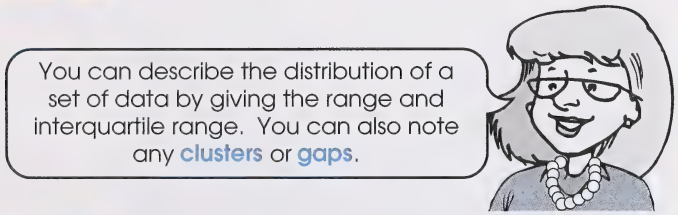


| | | | | | |
|---|---|---|---|---|---|
| 15 | 23 | 37 | 28 | 39 | 17 |
| 36 | 28 | 45 | 38 | 43 | 105 |
| 15 | 23 | 35 | 27 | 12 | 11 |
| 18 | 32 | 44 | 103 | 35 | 20 |
| 10 | 24 | 32 | 42 | 40 | 100 |

Give each of the following statistics:

**a.** interquartile range **b.** range

Check your answers by turning to the Appendix.




Clusters are several values grouped closely together. Gaps are spaces between values.

Stem and leaf plots and line plots are useful in finding the clusters and gaps in a set of data.

For example, examine the following stem and leaf plot for the set of data in Example 2.

| Stem | Leaf | |
|---|---|---|
| 3 | | |
| • | 6 | |
| 4 | | There is a gap between 36 and 59. |
| • | | |
| 5 | | |
| • | 9 | |
| 6 | 0, 1 | ← There is a gap between 61 and 70. |
| • | | |
| 7 | 0, 0, 3, 3, 4, 4 | Most of the data are clustered in this area. |
| • | 6, 6, 7, 7, 8, 9 | |
| 8 | 0, 0, 0, 2, 3, 3, 4 | |
| • | 5, 5, 5, 6, 6, 6, 6, 6, 7 | |
| 9 | 0, 0, 1, 2, 2, 3, 3 | |
| • | 6 | |

8. Examine the distribution of the data in question 6. Are there any clusters or gaps?

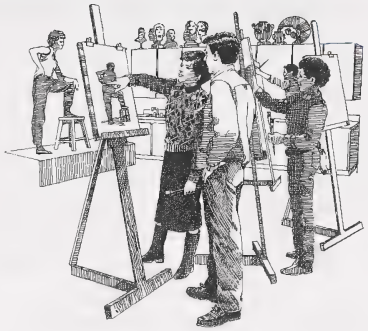
9. Examine the distribution of the data in question 7. Are there any clusters or gaps?

Check your answers by turning to the Appendix.

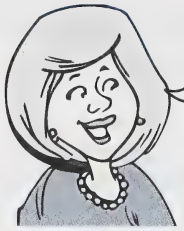
## Now Try This

Use a problem-solving strategy to answer the following question.

10. There are 24 students in Yasmin's art class. Of the students, 10 like to do oil paintings, 6 like to do water colours, and 4 like to do both. How many students like to do neither oil nor watercolour paintings?



Check your answer by turning to the Appendix.



The lower extreme, upper extreme, lower quartile, upper quartile, range, and interquartile range of a set of data give you useful information about how the data are distributed. It is also important to notice gaps and clusters in a set of data.

## Follow-up Activities

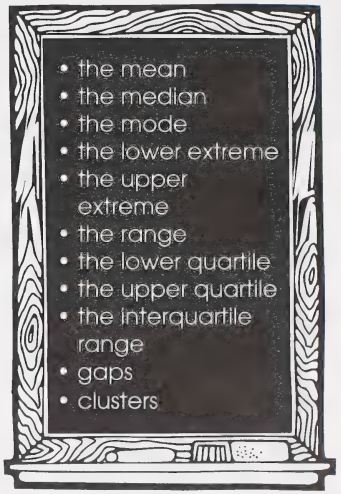
If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

## Extra Help

In this section you worked with data gathered by others. You used the terms listed at right to describe each set of data that you were given.

If you are unsure of the meaning of any of these terms, check the glossary or review the section activities.

Each of these terms represents a statistic.

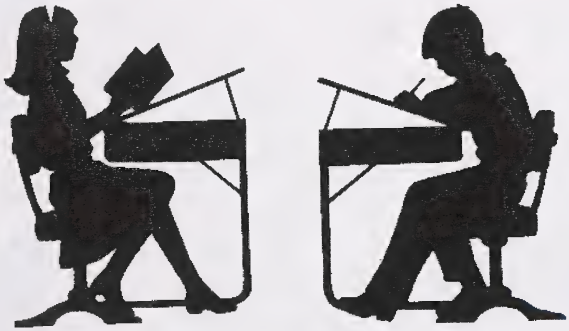


In order to calculate some of these statistics for a large set of data, you first organize the values using one of the following methods:

- line plots
- stem and leaf plots

Line plots and stem and leaf plots must be completed in a systematic manner, or you may miss a value or accidentally count a value twice. If you are having difficulty using these methods you could work with a partner.

One person should read aloud the values in the given set of data. The other person should record the values in a line plot or a stem and leaf plot.



The recorder may find it helpful to use 1-cm graph paper for the stem and leaf plot or the line plot. Graph paper helps to organize the data more neatly.

**1.** This is a list of the number of grams of fat in entrees (hamburgers, fishburgers, chickenburgers) sold in several fast-food restaurants.

| | | | |
|---|---|---|---|
| 49 | 44 | 46 | 29 |
| 34 | 50 | 29 | 36 |
| 27 | 27 | 38 | 50 |
| 16 | 25 | 29 | 64 |

Give each of the following statistics.

**a.** lower extreme **b.** upper extreme **c.** range
**d.** mean **e.** mode(s) **f.** median
**g.** lower quartile **h.** upper quartile **i.** interquartile range

**Hint:** First arrange the data in ascending order. To do this, work with a partner and make a stem and leaf plot. Use 1-cm graph paper to organize the data more neatly.

**2.** Examine the data in question 1. Are there any gaps? Are there any clusters?

Check your answers by turning to the Appendix.

Another method that you may use to organize the data is using a spreadsheet program such as *ClarisWorks*™.

## Example

This is a list of the pulse rates of 15 students.

| | | | | |
|---|---|---|---|---|
| 81 | 71 | 76 | 80 | 82 |
| 86 | 83 | 78 | 62 | 78 |
| 70 | 72 | 81 | 68 | 77 |

Use a spreadsheet program to write the numbers in order from least to greatest (ascending order).

## Solution

**Step 1:** Enter the data in the spreadsheet.

File Edit Format Calculate Options View

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 81 | | | | |
| 2 | 71 | | | | |
| 3 | 76 | | | | |
| 4 | 80 | | | | |
| 5 | 82 | | | | |
| 6 | 86 | | | | |
| 7 | 83 | | | | |
| 8 | 78 | | | | |

**Step 2:** Click on cell A1, hold, and drag to the bottom of the column of numbers so that all the numbers are highlighted, **or** click on cell A1 and then shift-click on cell A15.

File Edit Format Calculate Options View

A1 | 81

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 81 | | | | |
| 2 | 71 | | | | |
| 3 | 76 | | | | |
| 4 | 80 | | | | |
| 5 | 82 | | | | |
| 6 | 86 | | | | |
| 7 | 83 | | | | |
| 8 | 78 | | | | |

**Step 3:** Click and hold down on "Calculate" in the menu bar; a pull-down menu like this will appear.

File Edit Format **Calculate** Options View

81

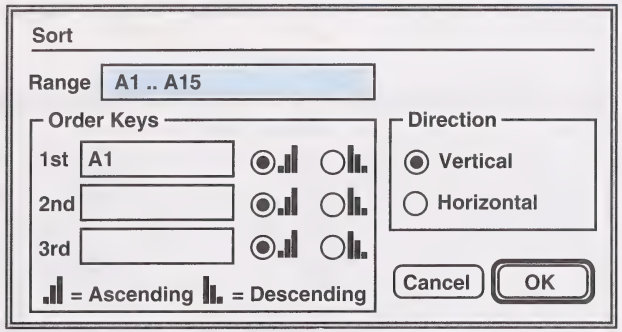
Move ...
Fill Right ⌘ R
Fill Down ⌘ D
Sort ... ⌘ J

Insert Cells ... ⇧⌘ I
Delete Cells ... ⇧⌘ K

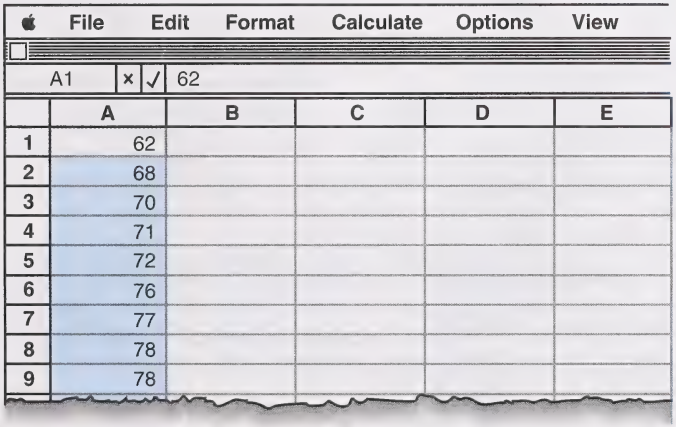
Calculate Now ⇧⌘ =
✓ Auto Calc

| | A | B | E |
|---|---|---|---|
| 1 | 81 | | |
| 2 | 71 | | |
| 3 | 76 | | |
| 4 | 80 | | |
| 5 | 82 | | |
| 6 | 86 | | |
| 7 | 83 | | |
| 8 | 78 | | |

Continue holding and drag down to "Sort." A dialogue box like this will appear.




**Step 4:** Make sure the options "Ascending" and "Vertical" in the dialogue box are selected. Click on the OK button or press "Return." The data will be arranged in ascending order.




**3.** This is a list of the number of days of snowfall for 24 Canadian cities in 1988.

| | | | |
|---|---|---|---|
| 40 | 65 | 31 | 48 |
| 61 | 100 | 59 | 56 |
| 78 | 51 | 53 | 83 |
| 108 | 65 | 36 | 6 |
| 71 | 67 | 47 | 52 |
| 41 | 65 | 43 | 3 |

Give each of the following statistics.

**a.** lower extreme **b.** upper extreme **c.** range
**d.** mean **e.** mode(s) **f.** median
**g.** lower quartile **h.** upper quartile **i.** interquartile range

**Hint:** First arrange the data in ascending order using a spreadsheet program such as *ClarisWorks*™.

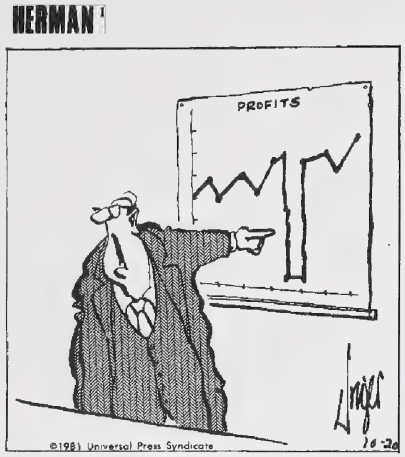
**4.** Examine the data in question 3. Are there any gaps in the set of data? Are there any clusters in the set of data?

Check your answers by turning to the Appendix.

## Enrichment

When reading data, think about any information you may have from outside mathematics; this knowledge can help you interpret data.

**1.** Use this cartoon to answer the following questions.




**"That's the last time I go on vacation."**

**a.** What kind of graph is pictured in the cartoon?

**b.** What do the vertical and horizontal axes represent?

**c.** What happened when the cartoon character was on vacation? Why might this situation have occurred?

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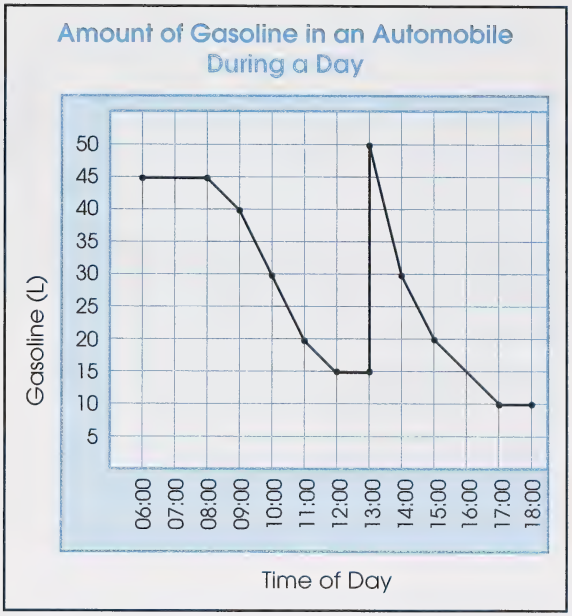
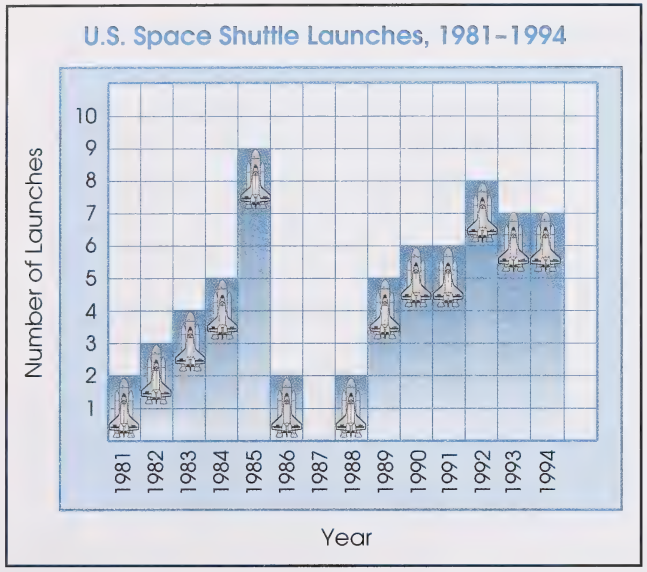
**2.** Use this broken-line graph to answer the following questions.




**a.** When did the driver of the car leave home?
**b.** When did the driver eat lunch?
**c.** When did the driver arrive home?
**d.** What do you think the driver does for a living? Why?
**e.** When did the driver purchase gas?
**f.** What is the capacity of the gas tank? **Hint:** Assume the driver filled the gas tank when he or she purchased gas.
**g.** How much gas did the driver purchase?
**h.** How much gas was left in the gas tank at 18:00?

**3.** Use this graph to answer the following questions.




**a.** Did the number of space shuttle launches increase or decrease from 1981 to 1985?

**b.** How many space shuttle launches were there in 1986? in 1987? in 1988?

**c.** What event could explain the decrease in the number of space shuttle launches in 1986, 1987, and 1988?

**4.** This a list of former presidents of the United States who were deceased as of December 1995, and the ages at which they died.

| | | |
|---|---|---|
| Washington ..... 67 | Fillmore ..... 74 | Roosevelt (T) ... 60 |
| Adams (J.) ..... 90 | Pierce ..... 64 | Taft ..... 72 |
| Jefferson ..... 83 | Buchanan ..... 77 | Wilson ..... 67 |
| Madison ..... 85 | Lincoln ..... 56 | Harding ..... 57 |
| Monroe ..... 73 | Johnson ..... 66 | Coolidge ..... 60 |
| Adams (J.Q.) ..... 80 | Grant ..... 63 | Hoover ..... 90 |
| Jackson ..... 78 | Hayes ..... 70 | Roosevelt (F.D.) 63 |
| Van Buren ..... 79 | Garfield ..... 49 | Truman ..... 88 |
| Harrison ..... 68 | Arthur ..... 57 | Eisenhower ..... 78 |
| Tyler ..... 71 | Cleveland ..... 71 | Kennedy ..... 46 |
| Polk ..... 53 | Harrison ..... 67 | Johnson ..... 64 |
| Taylor ..... 65 | McKinley ..... 58 | Nixon ..... 81 |

**a.** Make a stem and leaf plot using these stems: 4, 5, 6, 7, 8, 9.

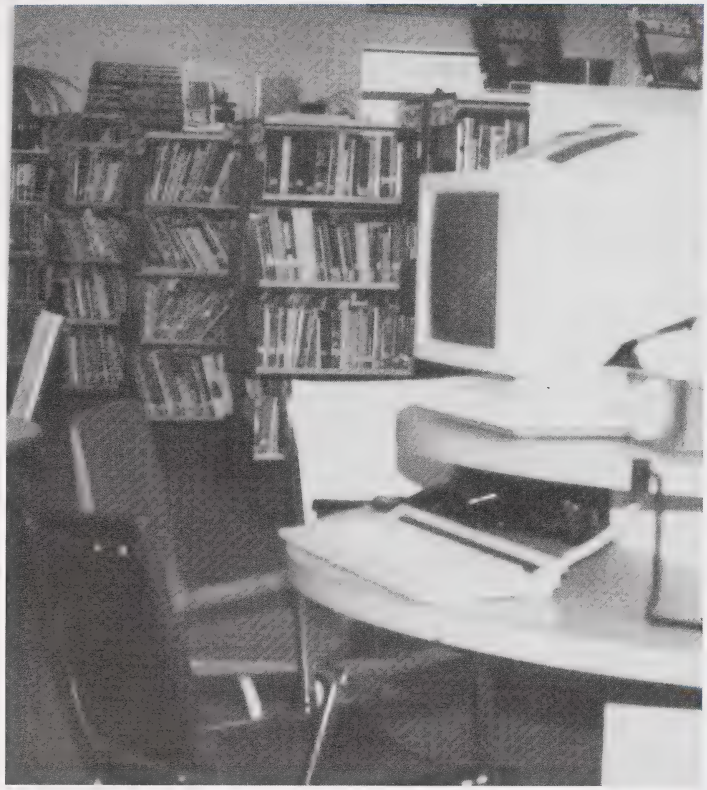
**b.** Which of these former presidents died in their eighties and nineties?

**c.** Which of these former presidents died in their forties or fifties?

**d.** Lincoln, Garfield, McKinley, and Kennedy all had the same cause of death. What was the cause of death of these former presidents?

Check your answers by turning to the Appendix.

## Conclusion



In this section you researched the answers to many questions that required numerical responses. You read tables and graphs. You analysed sets of data and found the mean, median, mode, lower and upper extremes, range, lower and upper quartiles, interquartile range, gaps, and clusters. You used these statistics to describe the distribution of a set of data.

In this section you may have used encyclopedias, atlases, newspapers, magazines, almanacs, books such as *The Guinness Book of Records*, computer databases such as *E-STAT* from Statistics Canada, and the Internet. Did you discover that some sources have more current data than others? Did you find some sources easier to use than others? Did you find any of the data surprising?

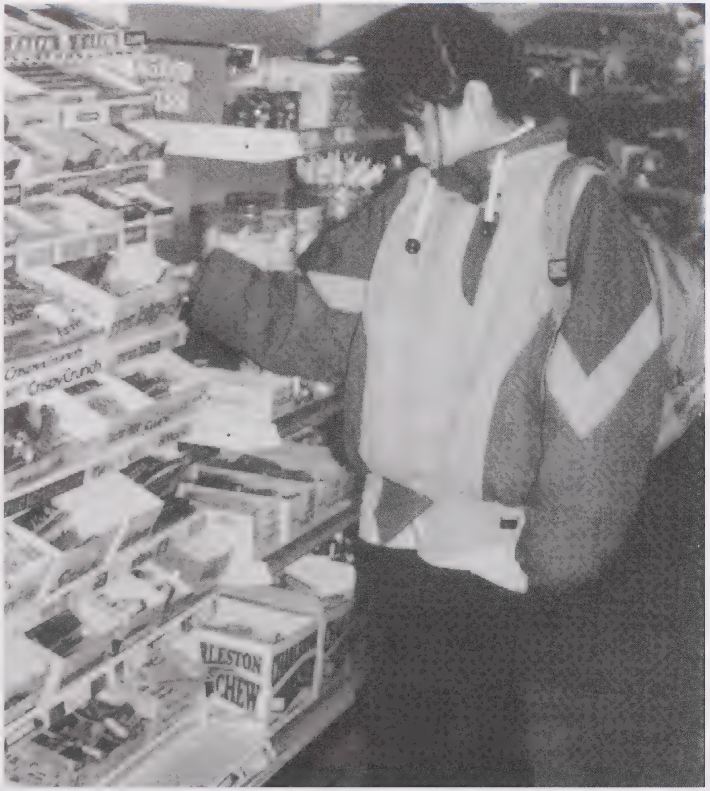
How do you think scientists have determined data such as the number of planets that orbit the Sun, the diameter of each planet, the distance each planet is from the Sun, and the number of satellites (moons) each planet has?

## Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 1.

# Section 2: First-Hand Data

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What is the favourite snack of teenagers? Is it chocolate bars, potato chips, popcorn, ice cream, or fruit?

To answer this question, you could conduct an opinion poll. But, for the results to be accurate, you must know about probability and how to collect first-hand data.

In this section you will use the probability formula, experiments, statistics, and the Monte-Carlo method to make predictions. You will also use surveys to describe a group of animals, plants, people, or things.

# Activity 1: What Is Probability?

Many people are afraid of lightning. What is the likelihood of your house being struck by lightning? Is it impossible, very unlikely, unlikely, likely, very likely, or certain?

The words *impossible*, *very unlikely*, *unlikely*, *likely*, *very likely*, and *certain* are used to describe the chance of something happening.

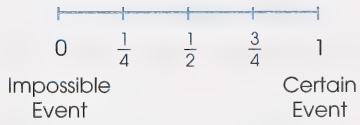
Another way to describe the likelihood of an event occurring is to give the **probability**.

Probability is a number from 0 to 1 that tells how likely an event is to happen.

If an event is **impossible**, the probability of the event occurring is 0. For example, it is impossible for anyone on Earth to float in the air for very long. So, the probability that this skier will stay airborne for an hour is 0.

If an event is **certain**, the probability of the event happening is 1. For example, if people go outside in the rain without umbrellas, they are certain to become wet. So, the probability that these people will become wet is 1.

Very few things in life are impossible or certain. This probability line shows that the probability of most events occurring is **between** 0 and 1.



**1.** Use the words *impossible*, *very unlikely*, *unlikely*, *likely*, *very likely*, or *certain* to describe the likelihood of an event happening with each of the following probabilities.

**a.** $\frac{1}{100}$ **b.** $\frac{1}{4}$ **c.** $\frac{3}{4}$ **d.** $\frac{99}{100}$

Check your answers by turning to the Appendix.

Sometimes a coin is flipped to decide which person will go first. For example, two tennis players could use a coin toss to decide who will serve first. Two golfers could use a coin toss to decide who will go up to the tee first.

When someone flips a coin there are two possible **outcomes**—"Heads" or "Tails."

Heads

Tails

The set of possible outcomes may be shown this way:

{Heads, Tails}

Each of the outcomes of the coin toss is **equally likely** to occur.

In statistics, an outcome is the result of an action, such as flipping a coin. Equally likely outcomes have the same chance of occurring.

Not all actions have equally likely outcomes. For example, when a tack is tossed, this is the set of possible outcomes:

{point up, point down}

Point up

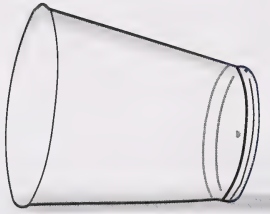
Point down

Because the base of the tack is heavier than the pointed end, there is a greater chance of the tack landing point up. The outcomes are **not** equally likely to occur.

**2.** Suppose this plastic glass is tossed.

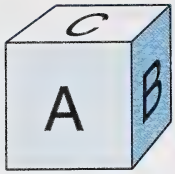
**a.** What are the possible outcomes?

**b.** Are the outcomes equally likely to occur? Why or why not?



Check your answers by turning to the Appendix.

3. This cube has six faces labelled A, B, C, D, E, and F. Suppose the cube is rolled.




   a. What are the possible outcomes? **Hint:** List the possible faces that can turn up.

   b. Are the possible outcomes equally likely to occur? Why or why not?

Check your answers by turning to the Appendix.

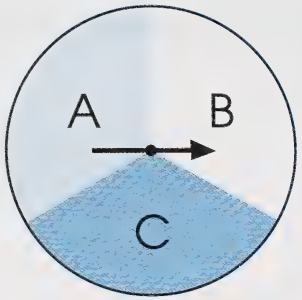
You have considered tossing a coin, a tack, and a glass. You have concluded that tossing a coin is fair; because the coin is evenly weighted, the possible outcomes are equally likely to occur. Tossing a tack or a glass is unfair; because the objects have irregular shapes, some outcomes are more likely to occur than others.

**Random** events are events that occur by chance and not influenced by outside factors.

One outside factor may be the shape of the object or the weight of the object. Another factor may be the person who performs the action.

For example, to ensure that a cube is rolled randomly, the cube must be evenly weighted. Also, you must shake the cube vigorously before rolling it so that you do not affect the outcome.

4. What rules could be made about twirling the spinner on this dial to ensure that the outcome is random?




Check your answers by turning to the Appendix.

## Theoretical Probability

When the possible outcomes of an action are **equally likely** to occur, you may use the **probability formula** to calculate the **theoretical probability** of an event.

The theoretical probability of an event equals the ratio of the number of **favourable outcomes** (what you want to happen) to the number of **possible outcomes** (what could happen).

The ratio is usually expressed in lowest terms. The theoretical probability of an event can be expressed by the following probability formula.

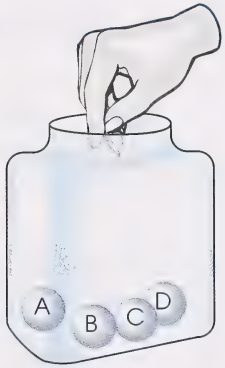
$$P(event) = \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

*P(event)* is read as "the probability of the event."

## Example 1

This jar contains four balls labelled A, B, C, and D. Suppose you choose a ball **at random** from the jar. (You mix the balls thoroughly and choose a ball without looking.)

What is the probability of choosing a ball labelled A?




## Solution

**Step 1:** List the set of possible outcomes and circle the favourable outcome(s).

**Hint:** Possible outcomes are what could happen; favourable outcomes are what you want to happen.

$\{(A), B, C, D\}$

**Step 2:** Count the number of favourable outcomes listed in Step 1.

There is 1 favourable outcome.

**Step 3:** Count the number of possible outcomes listed in Step 1.

There are 4 possible outcomes.

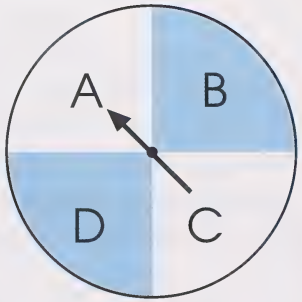
**Step 4:** Use the probability formula to calculate the probability that a ball labelled A will be chosen.

$$P(A) = \frac{1}{4} \qquad \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

$P(A)$ is read as "the probability of $A$."

The probability of choosing a ball labelled A is $\frac{1}{4}$.

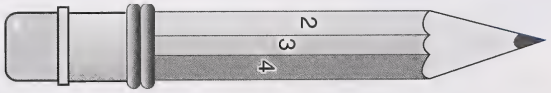
**5.** This dial has four sectors labelled A, B, C, and D. Suppose you twirl the spinner on the dial.




**a.** What is the probability of the spinner landing on a vowel?

**b.** What is the probability of the spinner landing a consonant?

**6.** This pencil has six faces labelled 1, 2, 3, 4, 5, and 6. Suppose you roll the pencil on a table.



**a.** What is the probability of rolling a 1?
**b.** What is the probability of rolling a prime number?
**c.** What is the probability of rolling a composite number?

Check your answers by turning to the Appendix.

In each of the events that you investigated so far, the possible outcomes have all been different. This is not always the case.

## Example 2

This dial has eight sectors. Four sectors are labelled 1, three sectors are labelled 5, and one sector is labelled 10. Suppose you twirl the spinner on the dial.


What is the probability of the spinner landing on a 5?

## Solution

**Step 1:** List the possible outcomes and circle the favourable outcomes.

**Hint:** Let the first sector labelled 1 be $1_1$, let the second sector labelled 1 be $1_2$, and so on.

$$\{1_1, 1_2, 1_3, 1_4, (5_1), (5_2), (5_3), 10\}$$

**Step 2:** Count the number of favourable outcomes listed in Step 1.

There are 3 favourable outcomes.

**Step 3:** Count the number of possible outcomes listed in Step 1.

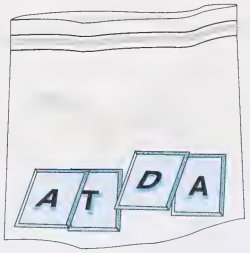
There are 8 possible outcomes.

**Step 4:** Use the probability formula to calculate the probability of landing on a 5.

$$P(5) = \frac{3}{8} \qquad \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of landing on a 5 is $\frac{3}{8}$.

**7.** This bag has four tiles labelled D, A, T, and A. Suppose a tile is chosen **at random** from the bag.



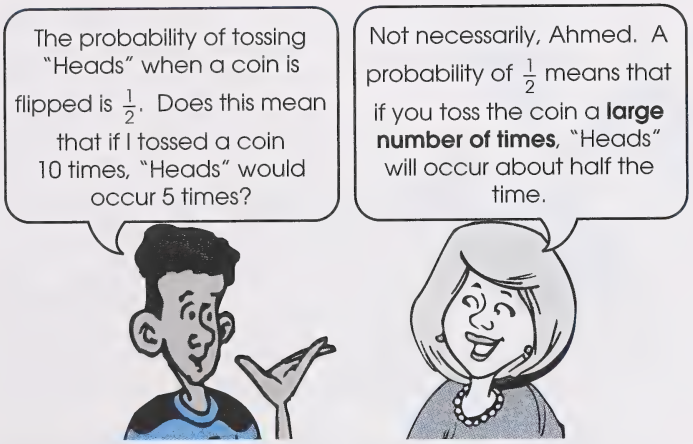

**a.** What is the probability of picking a tile labelled D?
**b.** What is the probability of picking a tile labelled A?
**c.** What is the probability of picking a tile labelled T?

8. Martin has a "loaded" coin. Both sides of the coin show a "Head." Martin tosses the coin.

   **a.** What is the probability that the coin will land "Heads" up?
   **b.** What is the probability that the coin will land "Tails" up?

Check your answers by turning to the Appendix.

## Experimental Probability




Ahmed tried this experiment. He tossed a coin 50 times and recorded the results, using tally marks, in a **frequency table**.

Frequency means the number of times a particular event occurred. A frequency table is a chart that lists a set of events together with the number of times each event occurred.

| Event | Tally | Frequency |
|---|---|---|
| Heads | 𝍸 𝍸 𝍸 𝍸 𝍸 II | 27 |
| Tails | 𝍸 𝍸 𝍸 𝍸 III | 23 |

In 50 trials, Ahmed found that "Heads" occurred 27 times. In other words, the **relative frequency** of "Heads" was $\frac{27}{50}$.

Relative frequency is the ratio of the frequency of a given event to the total number of trials. Relative frequency is also called **experimental probability**.

Ahmed then compared the relative frequency of tossing "Heads" in this experiment to the theoretical probability.

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{27}{50}$ | $\frac{1}{2} = \frac{25}{50}$ |

$$\frac{27}{50} \doteq \frac{1}{2}$$

Ahmed discovered that the relative frequency of "Heads" occurring was close to the theoretical probability of "Heads" occurring.

**9.** Check the probabilities that you calculated in question 5 of this activity by following these steps.

**Step 1:** Make a frequency table like this.

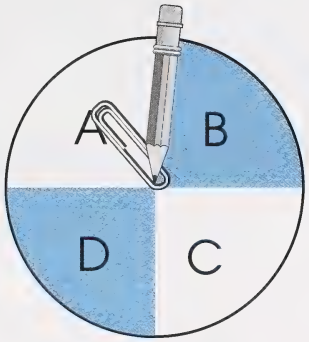
| Event | Tally | Frequency |
|---|---|---|
| a vowel | | |
| a consonant | | |

**Step 2:** Make a dial like the one in question 5 of this activity.

**Hint:** With a compass, draw a circle on a heavy sheet of paper or card stock. Use a protractor and straightedge to divide the circle into four sectors labelled A, B, C, and D.

Make a spinner using a sharpened pencil and a paper clip.

**Note:** Save the dial. You will need it again in Activity 2.




**Step 3:** Twirl the spinner 40 times and record the results in the frequency table.

**a.** Calculate the relative frequency of the spinner landing on a vowel. Is this close to the theoretical probability?

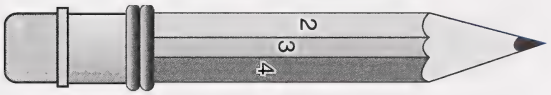
**b.** Calculate the relative frequency of the spinner landing on a consonant. Is this close to the theoretical probability?

**10.** Check the probabilities you calculated in question 6 of this activity by following these steps.

**Step 1:** Make a frequency table like this.

| Event | Tally | Frequency |
|---|---|---|
| 1 | | |
| a prime number | | |
| a composite number | | |

**Step 2:** Label the faces of a pencil 1, 2, 3, 4, 5, and 6.




**Step 3:** Roll the pencil on a table 60 times and record the results in the frequency table.

**a.** Calculate the relative frequency of rolling a 1. Is this close to the theoretical probability?

**b.** Calculate the relative frequency of rolling a prime number. Is this close to the theoretical probability?

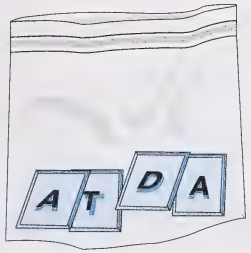
**c.** Calculate the relative frequency of rolling a composite number. Is this close to the theoretical probability?

**11.** Check the probabilities calculated in question 7 of this activity by following these steps.

**Step 1:** Make a frequency table like this.

| Event | Tally | Frequency |
|---|---|---|
| D | | |
| A | | |
| T | | |

**Step 2:** Print the letters D, A, T, and A on four identical tiles (squares of paper). Place these tiles in a bag.



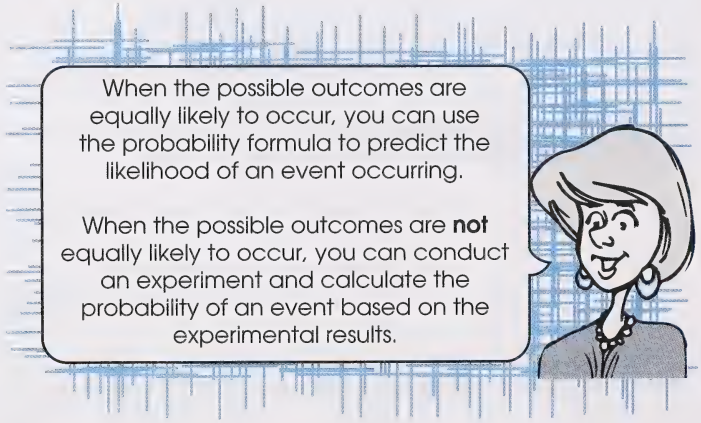

**Step 3:** Shake the bag to mix the tiles thoroughly; then, pick a tile, without looking, and record the results in the frequency table. Replace the tile and repeat this process for a total of 40 times.

**a.** Calculate the relative frequency of picking a tile labelled D. Is this close to the theoretical probability?

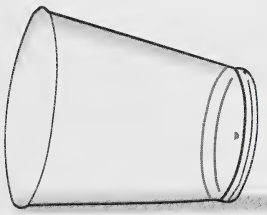
**b.** Calculate the relative frequency of picking a tile labelled A. Is this close to the theoretical probability?

**c.** Calculate the relative frequency of picking a tile labelled T. Is this close to the theoretical probability?

Check your answers by turning to the Appendix.



**12.** If you toss a plastic glass, what is the probability that it will land on its side?



Answer this question by doing an experiment. Toss a plastic glass 60 times, record the results in a frequency table, and then calculate the probability of the cup landing on its side.

**13.** If you toss a thumbtack, what is the probability that it will land point up?

Answer this question by doing an experiment. Toss a tack 60 times, record the results in a frequency table, and then calculate the probability of the tack landing point up.

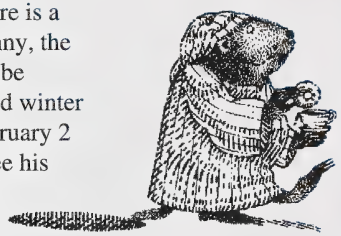
Check your answers by turning to the Appendix.

## Statistical Probability

There is another technique that you can use to predict the likelihood of an event happening—studying past data.

## Example

February 2 is Groundhog Day. There is a legend that says if February 2 is sunny, the groundhog will see his shadow and be frightened back into hibernation, and winter will last for six more weeks. If February 2 is cloudy, the groundhog will not see his shadow and there will be an early spring.



Records for a particular town show that it has been sunny on February 2 in 17 out of the past 50 years.

What is the probability that it will be sunny there on February 2 next year?

## Solution

The town had sunny weather on February 2 in 17 out of 50 years. In other words, the relative frequency of sunny days on February 2 was $\frac{17}{50}$.

$$\therefore\ P(\text{sunny}) = \frac{17}{50}$$

$$\frac{\text{Number of Times the Day Was Sunny}}{\text{Total Number of Days Researched}}$$

Based on past data, the probability that it will be sunny on the next February 2 is about $\frac{17}{50}$.

**14.** The Gryphons basketball team and a rival team have played each other five times this season. The Gryphons won 4 of the 5 games. When the teams next meet in a basketball game, what is the probability that the Gryphons will win?

**15.**

Dakota has made 60% of his free-throw shots this season. If Dakota is fouled in a game and awarded a foul shot, what is the probability that he will be successful in making the shot?

Check your answers by turning to the Appendix.

## Did You Know?

In 1563, the Italian mathematician Girolamo Cardano, also known as Jerome Cardan, wrote the first book on the topic of probability.

Two seventeenth-century French mathematicians are also recognized for their pioneer work on the theory of probability—Pierre de Fermat and Blaise Pascal.

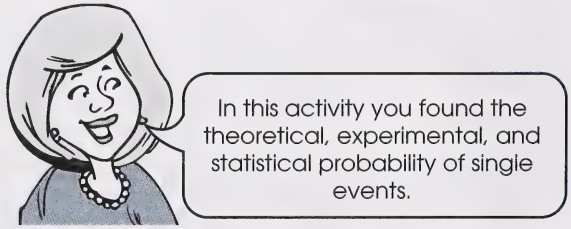
Use the Internet to discover more about Cardan, Fermat, and Pascal.

## Now Try This

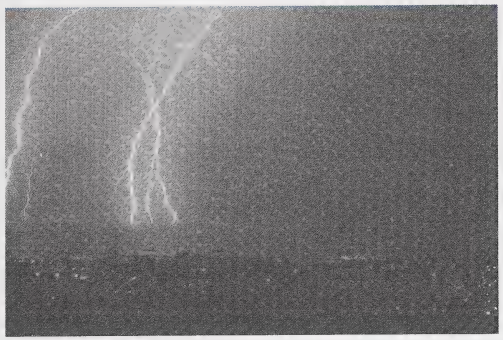
Use a problem-solving strategy to answer the following question.

**16.** Amber is taller than Marcia. Peggy is taller than Amber but not as tall as Oksana. Marcia is shorter than Peggy but not as short as Roxanne. List the students from tallest to shortest.

Check your answer by turning to the Appendix.



# Activity 2: Probability of Two or More Independent Events



Between 1942 and 1977, former park ranger Roy Sullivan of the United States was struck by lightning seven times—more times than any other person. As a result, his name appears in *The Guinness Book of Records*.

What do you think the probability is of being struck by lightning once? twice? three times? four times? five times? six times? seven times?

In Activity 1, you examined the probability of single events. In this activity you will investigate the probability of two or more **independent events**.

Independent events are events that do not depend on each other. Independent events can happen **together** or one **after** the other.

You can use graph paper or a table to list the possible outcomes of **two** independent events.

## Example 1

Suppose you have a nickel and a dime. When you toss the nickel, you will get either "Heads" or "Tails." The same is true for the dime.

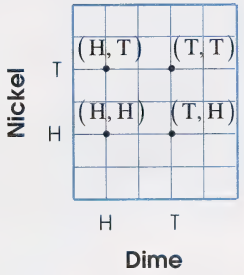
When you toss the nickel and the dime at the same time, what are the possible outcomes?

## Solution

### Method 1: Using Graph Paper

Let H be "Heads" and T be "Tails."

Make a graph like this. Along the $x$-axis, label the possible outcomes for one coin. Along the $y$-axis, label the possible outcomes for the other coin. Indicate the possible outcomes for both coins by putting dots on the graph and naming the ordered pairs.




This is the set of possible outcomes when both coins are tossed:

$$\{(H, H),\ (H, T),\ (T, H),\ (T, T)\}$$

## Method 2: Using a Table

Let H be "Heads" and T be "Tails."

Make a table like this. Along the rows of the table, label the possible outcomes for one coin. Along the columns of the table, label the possible outcomes for the other coin. In the body of the table, indicate the possible outcomes for both coins.

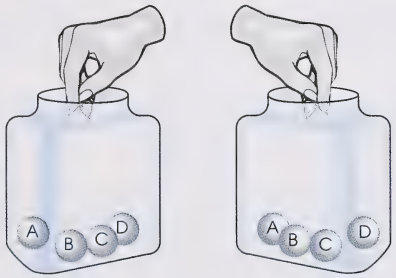
| Dime \ Nickel | H | T |
|---|---|---|
| H | (H, H) | (H, T) |
| T | (T, H) | (T, T) |

This is the set of possible outcomes when both coins are tossed:

$$\{(H,H),\ (H,T),\ (T,H),\ (T,T)\}$$

Use graph paper or a table to answer questions 1 and 2.

**1.** Each of these jars has balls labelled A, B, C, and D.




Suppose you mix the balls thoroughly and, without looking, pick a ball from each jar. What are the possible outcomes?

**2.** You have two cubes. The faces of each cube are labelled 1, 2, 3, 4, 5, and 6. Suppose you roll the two cubes.

What are the possible outcomes? **Hint:** List the possible faces that can turn up.

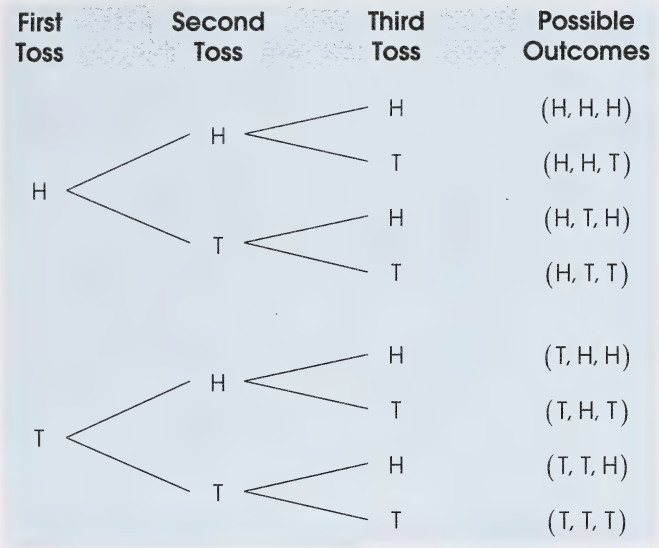
Check your answers by turning to the Appendix.

## Example 2

Suppose you toss a coin three times.
What are the possible outcomes?

## Solution

Make a tree diagram. Let H be "Heads." Let T be "Tails."

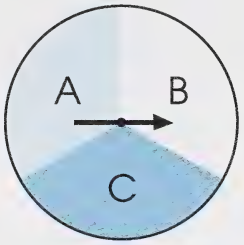


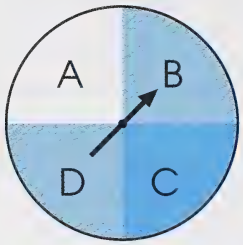

This is the set of possible outcomes:

$$\{(H,H,H),\ (H,H,T),\ (H,T,H),\ (H,T,T),\ (T,H,H),$$
$$(T,H,T),\ (T,T,H),\ (T,T,T)\}$$

Use tree diagrams to answer questions 3 and 4.

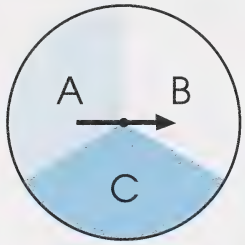
**3.** The first dial has three sectors labelled A, B, and C. The second dial has four sectors labelled A, B, C, and D. Suppose you twirl the spinner on the first dial and then you twirl the spinner on the second dial.



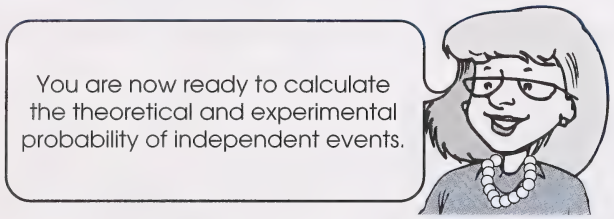

What are the possible outcomes?

**4.** Suppose you twirl the spinner on this dial and then toss a coin.

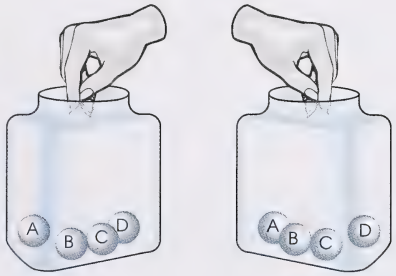



What are the possible outcomes?

Check your answers by turning to the Appendix.




**5.** You have two jars. Each of the jars has balls labelled, A, B, C, and D.




Suppose you mix the balls thoroughly and, without looking, pick a ball from each jar.

**Note:** You listed the possible outcomes for this situation in question 1 of this activity.

**a.** What is the probability of choosing two vowels?

**b.** What is the probability of choosing two consonants?

**c.** What is the probability of choosing a vowel and a consonant?

**6.** Check the theoretical probabilities that you calculated in the previous question (question 5) by following these steps.

**Step 1:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| two vowels | | |
| two consonants | | |
| vowel and consonant | | |

**Step 2:** Gather two jars and eight ping-pong balls. Label four balls A, B, C, and D and put them in one jar. Label the remaining four balls A, B, C, and D and put them in the other jar. **Note:** If you don't have ping-pong balls, use styrofoam balls, sugar cubes, or tiles of paper.

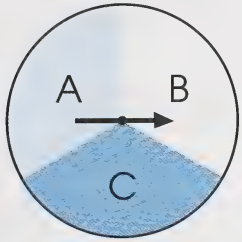
**Step 3:** Mix the balls thoroughly and, without looking, pick a ball from each jar. Record the results in the frequency table. Replace the balls and repeat the process for a total of 48 times.

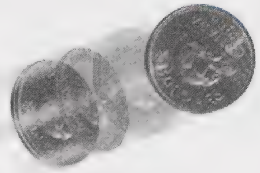
**a.** Calculate the relative frequency of picking two vowels. Is this close to the theoretical probability?

**b.** Calculate the relative frequency of picking two consonants. Is this close to the theoretical probability?

**c.** Calculate the relative frequency of picking a vowel and a consonant. Is this close to the theoretical probability?

**7.** Suppose you twirl the spinner on this dial and then toss a coin.



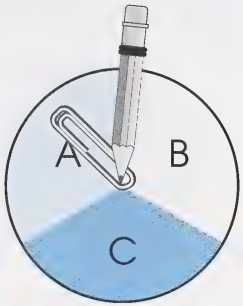
**Hint:** You listed the possible outcomes of this situation in question 4 of this activity.

**a.** What is the probability of landing on a vowel and then tossing "Heads"?

**b.** What is the probability of landing on a consonant and then tossing "Heads"?

**8.** Check the theoretical probabilities you calculated in the previous question (question 7) by following these steps.

**Step 1:** Use a compass to make a circle on heavy paper or card stock. Use a protractor and straightedge to divide the circle into three congruent sectors. Label the sectors like the dial in question 7. Make a spinner using a sharpened pencil and a paper clip.




**Step 2:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| a vowel and "Heads" | | |
| a consonant and "Heads" | | |
| a vowel and "Tails" | | |
| a consonant and "Tails" | | |

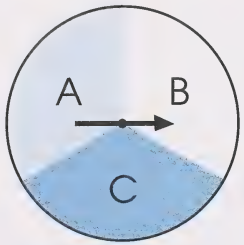
**Step 3:** Gather a coin and the dial you made in Step 1. Twirl the spinner on the dial; then toss the coin. Record the results in the frequency table. Repeat this process for a total of 36 times. **Note:** Save the dial. You will need it for a later question.

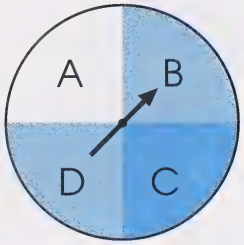
**a.** Calculate the relative frequency of landing on a vowel and then tossing "Heads." Is this close to the theoretical probability?

**b.** Calculate the relative frequency of landing on a consonant and then tossing "Heads." Is this close to the theoretical probability?

You may wish to calculate the other relative frequencies on your own.

**9.** The first dial has three sectors labelled A, B, and C. The second dial has four sectors labelled A, B, C, and D.




Suppose you twirl the spinner on the first dial and then you twirl the spinner on the second dial.

**Hint:** You listed the possible outcomes of this situation in question 3 of this activity.

**a.** What is the probability of landing on two vowels?

**b.** What is the probability of landing on two consonants?

**c.** What is the probability of landing on a vowel and a consonant?

**10.** Check the theoretical probabilities you calculated in the previous question (question 9) by following these steps.

**Step 1:** Gather the two dials that you made in this module. Make a spinner for each dial by using a sharpened pencil and a paper clip.

**Step 2:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| two vowels | | |
| two consonants | | |
| a vowel and a consonant | | |

**Step 3:** Twirl the spinner on the first dial; then twirl the spinner on the second dial. Record the results in the frequency table. Repeat this process for a total of 36 times.

**a.** Calculate the relative frequency of landing on two vowels. Is this close to the theoretical probability?

**b.** Calculate the relative frequency of landing on two consonants. Is this close to the theoretical probability?

**c.** Calculate the relative frequency of landing on a vowel and a consonant. Is this close to the theoretical probability?

Check your answers by turning to the Appendix.

Did you find that with more trials, the relative frequency of an event is closer to the theoretical probability?

**11.** You have two cubes. The faces of each cube are labelled 1, 2, 3, 4, 5, and 6.

Suppose you roll these two cubes.

**Hint:** You listed the possible outcomes of this situation in question 2 of this activity.

**a.** What is the probability of rolling two identical numbers?

**b.** What is the probability of rolling two numbers with a sum of 7?

**12.** Check the theoretical probabilities that you calculated in the previous question (question 11) by following these steps.

**Step 1:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| two identical numbers | | |
| two numbers with a sum of 7 | | |
| other | | |

**Step 2:** Gather two sugar cubes and label the faces of the cubes 1, 2, 3, 4, 5, and 6.

**Note:** If you do not have sugar cubes, you may use a pair of standard dice. Let ⚀ be 1, ⚁ be 2, ⚂ be 3, ⚃ be 4, ⚄ be 5, and ⚅ be 6.

**Step 3:** Toss the cubes 36 times and record the results in the frequency table.

**a.** Calculate the relative frequency of rolling two identical numbers. Is this close to the theoretical probability?

**b.** Calculate the relative frequency of rolling two numbers with a sum of 7. Is this close to the theoretical probability?

Check your answers by turning to the Appendix.

## Did You Know?

The **law of large numbers** states that the larger the number of trials you conduct, the closer the experimental results will be to the theoretical results.

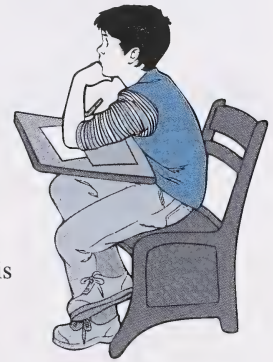

The law of large numbers was first stated by Jacques (also referred to as James, Jacob, Jakob) Bernoulli (1654–1705). Use the Internet to discover more about Jacques Bernoulli.

# Monte-Carlo Method

Now you will use the **Monte-Carlo method** to solve problems.

The Monte-Carlo method is a technique where an event is modelled using probability tools such as coins, dice, and dials. The model may be called a **simulation**.

## Example 1



Thor has to take a true/false test with 100 questions. Suppose he simply guesses at the answer to each question. Predict how many questions Thor will get correct.

Design a model that will help you solve this problem.

## Solution

**Step 1:** Use this reasoning to design a model that will help you solve this problem.

On each question of a true/false test there are two possible outcomes; one outcome is right and one outcome is wrong. Therefore, you can use a coin to model the problem. Let "Heads" represent a correct answer and "Tails" represent a wrong answer.

There are 100 questions on the test, so the coin must be tossed 100 times. Each toss simulates answering a question by guessing.

**Step 2:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| Heads | | |
| Tails | | |

**Step 3:** Toss a coin 100 times and record the events in the frequency table.

**Step 4:** Count the number of "Heads" tossed. This is an estimate of what Thor's score on the true/false test will be if he guesses at each answer.

## Example 2

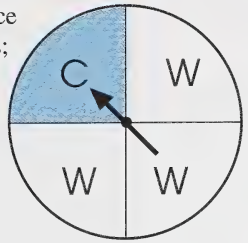
Jessica has to take a multiple-choice test. The test has 100 questions and each question has four answers from which to choose. Suppose Jessica simply guesses at the answer to each question. Predict how many questions she will get right.

Design a model to help you solve this problem.

## Solution

**Step 1:** Use this reasoning to design a model that will help you solve this problem.

On each question of a multiple-choice test there are four possible outcomes; three outcomes are wrong and one is right. Therefore, you can use a dial like this to model the problem.




**Note:** Let C be a correct answer; let W be a wrong answer.

There are 100 questions on the test, so the spinner must be twirled 100 times. Each twirl simulates answering a question by guessing.

**Step 2:** Make a frequency table like this:

| Event | Tally | Frequency |
|---|---|---|
| C | | |
| W | | |

**Step 3:** Make a dial like the one shown in step 1. Make a spinner using a sharpened pencil and a paper clip.

**Step 4:** Twirl the spinner 100 times and record the results in the frequency table.

**Step 5:** Count the number of times the spinner landed on C. This is an estimate of what Jessica's score on the multiple-choice test will be if she guesses at each answer.

**13. a.** Solve the problem in Example 1 (Thor's true/false test) by following the steps given.

**b.** Solve the problem in Example 2 (Jessica's multiple-choice test) by following the steps given.

**14.** Suppose a breakfast cereal company has randomly placed one of six baseball cards in each box of cereal it manufactures. Predict how many cereal boxes you will need to purchase in order to obtain a complete set of cards.


**a.** Design a model to solve this problem.
**b.** Solve the problem.

Check your answers by turning to the Appendix.

## Did You Know?

Stanislow Marcin Ulam (1909–1984) devised the Monte-Carlo method.


Use the Internet to discover more about Ulam.

## Now Try This

Use a problem-solving strategy to answer the following question.

**15.** When 18 bottles were placed in a case like the one shown in the diagram, every row and every column contained an even number of bottles.

Draw a similar diagram. Show a possible arrangement of the bottles in the case by putting ✗s in the diagram.

Check your answer by turning to the Appendix.

In this activity you found the theoretical and experimental probabilities of independent events. You used the Monte-Carlo method to solve probability problems.

## Activity 3: Conducting Surveys

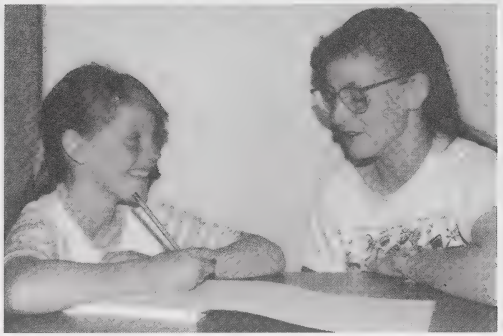
Before Radnia begins to read a chapter of a textbook, she takes a few minutes to survey the chapter. She reads the headings in the chapter and examines the diagrams and photographs. This survey helps her to pick out the main ideas of the chapter.

Researchers use **statistical surveys** to help them find out more about a set of people, animals, or things.

A statistical survey is a collection of data that describes some aspect of a set of people, animals, or things. The set of people, animals, or things being studied is called the **population**.

There are different kinds of surveys.

- In an opinion survey (often called a **poll**), people are asked questions and their responses are recorded. Then the different responses are counted.



- In a traffic survey, the vehicles on a particular road are counted.

- In a **population** survey, the total number of plants, animals, or people are counted. For example, the number of whooping cranes in Canada and the United States have been counted since 1941 when only 15 wild whooping cranes remained.

WOOD BUFFALO NATIONAL PARK/R. D. MUIR, CWS


Explore the Internet or visit your local library to find the answers to question 1.

**1.** The largest survey of people in Canada is the **Census**.

**a.** How is the census of Canadians conducted?
**b.** How often is the census conducted?
**c.** How much money did the latest census cost?
**d.** Why is the census conducted?

Check your answers by turning to the Appendix.

A survey of a large population may take a long time and can be very expensive. To save time or money, a **sample** of the population may be studied instead.

A sample is a part of the population used to obtain information about the whole group.

When you choose a sample, you need to keep the following suggestions in mind:

- The sample should be as large as possible.
- The sample should be representative of the population. For example, if the target population is teenagers, then the sample survey should be a group of teenagers.
- The members of the sample should be chosen at random. That is, everything or everyone in the population should have the same chance of being chosen.

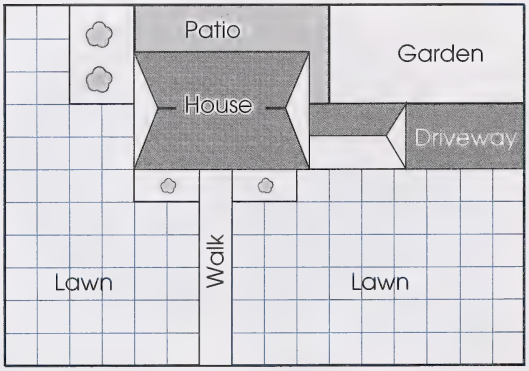
If these suggestions are followed, you can use the sample results to make estimates or predictions about the whole group.

## Example 1

Stephen wanted to estimate the number of dandelions, broad-leaf plantains, and clover plants in his lawn.

Here are the steps Stephen used to solve this problem.

**Step 1:** To make the counting easier, Stephen used string and posts to divide the lawn into 100 equally sized squares.



**Step 2:** Stephen randomly selected a sample of 20 squares. He then identified and counted the weeds in this sample. He recorded the data in a frequency table like this.

| Kind of Weed | Tally | Frequency |
|---|---|---|
| Dandelions | 𝍸 𝍸 𝍸 | 15 |
| Broad-leaf Plantains | 𝍸 I | 6 |
| Clover Plants | 𝍸 𝍸 II | 12 |

**Step 3:** Stephen used proportional reasoning to estimate the total number of each kind of weed in the entire lawn.

- Dandelions

$$\frac{15}{20} \overset{\times 5}{=} \frac{75}{100}$$

$$\frac{\text{number of dandelions}}{\text{number of squares}}$$

- Broad-leaf Plantains

$$\frac{6}{20} \overset{\times 5}{=} \frac{30}{100}$$

$$\frac{\text{number of broad-leaf plantains}}{\text{number of squares}}$$

- Clover Plants

$$\frac{12}{20} \overset{\times 5}{=} \frac{60}{100}$$

$$\frac{\text{number of clover plants}}{\text{number of squares}}$$

Stephen estimated that there were 75 dandelions, 30 broad-leaf plantains, and 60 clover plants in the entire lawn.

## Example 2

The student council at Northwood Junior High had to make a decision about what colour the school jackets should be. They had a choice of four colours—blue, green, gold, and garnet. Before making a decision, the student council wanted to predict what colour of school jacket the students would prefer.

Here are the steps the student council used to solve the problem.

**Step 1:** The student council decided on the survey question:

What colour of school jacket do you prefer—blue, green, gold, or garnet?

**Step 2:** The student council randomly selected 20 of the 180 students to interview. They asked each of these students the question. The council recorded the responses in a frequency table like this.

| Colour | Tally | Frequency |
|---|---|---|
| Blue | // | 2 |
| Green | 𝍸 | 5 |
| Gold | 𝍸 / | 6 |
| Garnet | 𝍸 // | 7 |

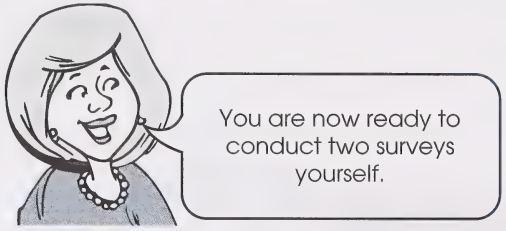
**Step 3:** The student council used proportional reasoning to calculate the percent frequency for each colour.

**Note:** The percent frequency is the relative frequency in percent form.

| Colour | Tally | Frequency | Percent Frequency |
|---|---|---|---|
| Blue | // | 2 | 10% |
| Green | 𝍸 | 5 | 25% |
| Gold | 𝍸 / | 6 | 30% |
| Garnet | 𝍸 // | 7 | 35% |

**Step 4:** The student council used the survey results to make a decision.

Based on the survey, the student council predicted that the most popular colour for the school jackets would be garnet.




**2.** Conduct Survey 1 by following these steps.

**Step 1:** Make a frequency table like this:

| Number | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

**Step 2:** Randomly select a sample of 10 people to survey.

**Step 3:** One at a time, show each person the following list of numbers:

1 2 3 4

Ask each person to pick one of the four numbers. Record each person's choice in the frequency table.

**a.** Calculate the percent frequency for each number.

**b.** Based on the survey results, predict which number will be picked by the next person you ask.

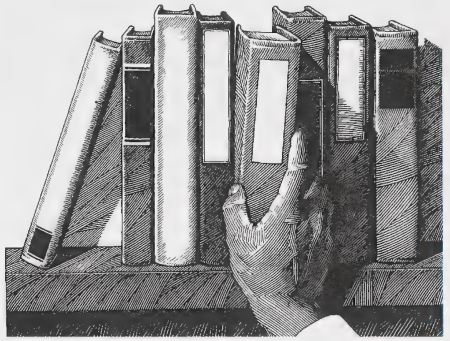
**c.** Did the results of the survey surprise you? Why or why not?

**3.** Conduct Survey 2 by following these steps.

**Step 1:** Make a frequency table like this. List each letter of the alphabet.

| Letter | Tally | Frequency |
|---|---|---|
| a | | |
| b | | |
| c | | |
| d | | |
| e | | |

**Step 2:** At random, select a novel (published in English) from a bookshelf and open the book to any page.



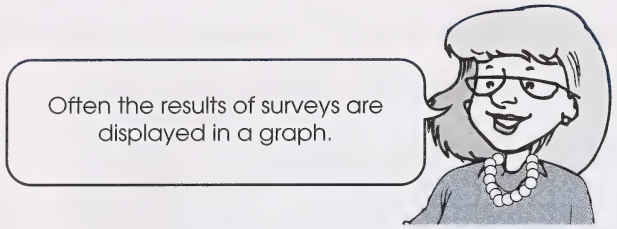
**Step 3:** Locate the first 25 words on the page and count how many times each letter of the alphabet occurs in these 25 words. Record the results in the frequency table.

**Hint:** Look at the first letter and put a tally in the appropriate row; then look at the second letter and put a tally in the appropriate row, and so on.

**a.** Based on the survey results, determine the two most frequently used vowels in English.

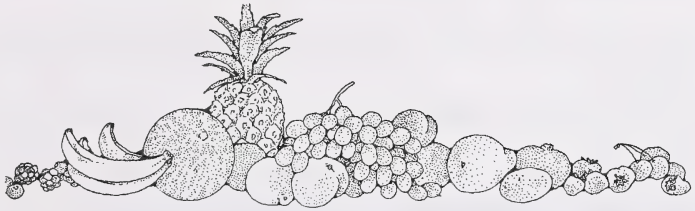
**b.** Based on the survey results, determine the two most frequently used consonants in English.

Check your answers by turning to the Appendix.




## Example 3

Thach wanted to determine the different kinds of fruit in a can of fruit cocktail, and the relationship of these with respect to quantity. He wanted to display this information in a circle graph.



Following are the steps he used to solve this problem.

**Step 1:** Thach selected at random a can of fruit cocktail from the shelf. He shook the can, opened it, and took out a scoop of fruit cocktail.

**Note:** The scoop is a representative sample of the whole. Therefore, Thach reasoned that the fruit in the can will be in the same proportion as the fruit in the scoop.

**Step 2:** Thach counted the number of pieces of each fruit in the sample. He recorded the results in a frequency table.

| Fruit | Tally | Frequency |
|---|---|---|
| Apples | 𝍸 𝍸 III | 13 |
| Bananas | 𝍸 I | 6 |
| Grapes | II | 2 |
| Peaches | 𝍸 𝍸 I | 11 |
| Pears | IIII | 4 |

**Step 3:** Thach used proportional reasoning to calculate how many degrees of the circle graph each fruit should be. **Note:** There were 36 pieces of fruit. A circle has 360°.

Apples: $\frac{13}{36} = \frac{130}{360}$ ($\times 10$)

Bananas: $\frac{6}{36} = \frac{60}{360}$ ($\times 10$)

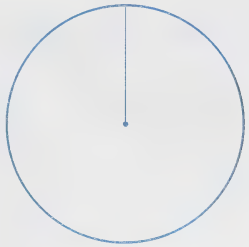
Grapes: $\frac{2}{36} = \frac{20}{360}$ ($\times 10$)

Peaches: $\frac{11}{36} = \frac{110}{360}$ ($\times 10$)

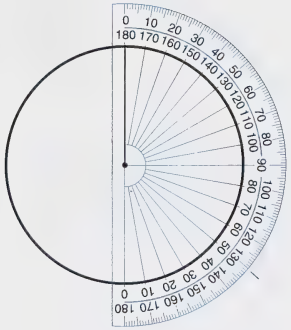
Pears: $\frac{4}{36} = \frac{40}{360}$ ($\times 10$)

**Step 4:** Thach drew a circle and a radius with a compass and straightedge.

**Note:** See Module 4 to review the meaning of radius.

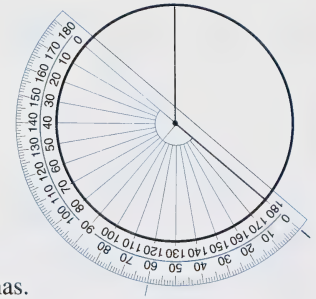


**Step 5:** Thach placed a protractor on the diagram so that the base line of the protractor was on the radius of the circle and the centre of the protractor was on the centre of the circle. Then Thach made a mark opposite 130°, the number of degrees needed to represent the apples. (See Step 3.)



**Step 6:** Thach removed the protractor and used a straightedge to complete the angle for the apples sector.

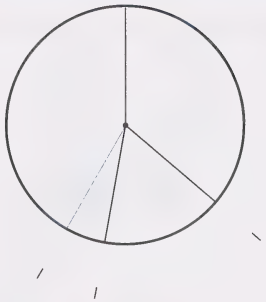
**Step 7:** Thach placed a protractor on the diagram so that the base line of the protractor was on the ray of the angle just constructed and the centre of the protractor was on the centre of the circle. He then made a mark opposite 60°, the number of degrees needed to represent the bananas. (See Step 3.)



**Step 8:** Thach removed the protractor and used a straightedge to complete the angle for the bananas sector.

**Step 9:** Thach placed the protractor on the diagram so that the base line of the protractor was on the ray of the angle just constructed and the centre of the protractor was on the centre of the circle. He then made a mark opposite 20°, the angle needed to represent the grapes. (See Step 3.)

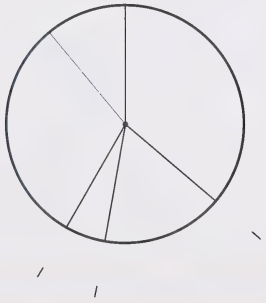
**Step 10:** Thach removed the protractor and used a straightedge to complete the angle for the grapes sector.



**Step 11:** Thach placed the protractor on the diagram so that the base line of the protractor was on the ray of the angle just constructed and the centre of the protractor was on the centre of the circle. He then made a mark opposite 110°, the angle needed to represent the peaches. (See Step 3.)

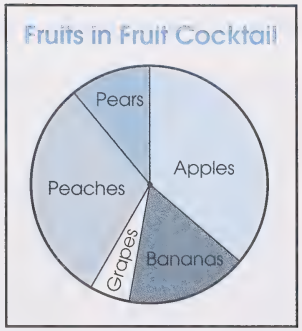
**Step 12:** Thach removed the protractor and used a straightedge to complete the angle for the peaches sector.

The remaining sector represents the pears.



**Step 13:** Thach labelled the circle graph and gave it a title.

This circle graph shows the kinds of fruit and the relative amount of each.




**4.** Ruby selected 30 students at random to answer this question:

What is your favourite team sport—basketball, football, hockey, soccer, or volleyball?

Ruby recorded the responses in a frequency table like this.

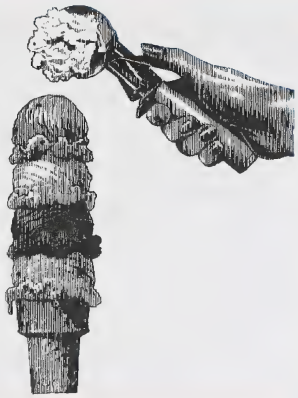
| Favourite Team Sport | Tally | Frequency |
|---|---|---|
| basketball | 𝍸 𝍸 | 10 |
| football | 𝍸 | 5 |
| hockey | 𝍸 \|\| | 7 |
| soccer | \|\|\| | 3 |
| volleyball | 𝍸 | 5 |

Display this data in a circle graph.

**5.** Frieda selected 40 elementary students at random and asked them this question:

What is your favourite flavour of ice cream—bubble gum, chocolate, maple walnut, strawberry, or vanilla?

Frieda recorded the responses in a frequency table.



| Favourite Flavour of Ice Cream | Tally | Frequency |
|---|---|---|
| bubble gum | 𝍸 IIII | 9 |
| chocolate | 𝍸 𝍸 𝍸 | 15 |
| maple walnut | II | 2 |
| strawberry | 𝍸 III | 8 |
| vanilla | 𝍸 I | 6 |

Display this data in a circle graph.

Check your answers by turning to the Appendix.

## Sampling Techniques in Opinion Surveys

How do television networks know what programs are the most popular? Canadian television networks rely upon the data provided by two organizations—the A. C. Nielsen Company and the Bureau of Broadcast Measurement.

The A. C. Nielsen Company monitors the television viewing habits of Canadians by randomly selecting 1500 households and placing "people meters" in these homes. The people meters provide data on what programs are viewed and who is watching them.


Use the Internet to discover more about these people meters. This is the uniform resource locator (URL) of a site that you may find helpful.

http://www.nielsenmedia.com/company/history.htm

The Bureau of Broadcast Measurement (BBM) monitors the television viewing habits of Canadians by conducting four surveys per year—pre-fall, fall, spring, and summer. The fall and spring surveys are the largest. BBM telephones randomly selected households and asks them to participate in their survey. Each family that agrees to participate is sent a diary for each working television set. Family members complete the diaries for a three-week period and then the diaries are returned to BBM.

Each year, Canadians are surveyed on many other topics. Four types of sampling techniques are commonly used.

- **Personal Interview:** Trained interviewers administer a questionnaire in person. The advantage of a personal interview is that this kind of survey is taken quite seriously and there usually is a good response rate. The disadvantage is that if people are not at home, the interviewer may need to make a second visit. Interviewing the participants in person is very time-consuming and expensive.

- **Telephone Survey:** Trained interviewers administer a questionnaire over the telephone. The advantage of a telephone survey is that it is usually easier to contact people. If people are not at home, the call can be repeated easily at a later time. However, people decline to participate in a telephone survey more than in a personal interview.

- **Mail Survey:** A questionnaire is mailed to selected individuals. This method of surveying is less costly because people do not have to be trained to administer the questionnaire. However, mail surveys are often viewed as "junk mail" and are discarded, so fewer people participate in the poll.

- **Other Surveys:** A questionnaire is placed in a restaurant, hotel, or business where clients may notice them. Questionnaires are also placed in magazines, newsletters, books, and on the Internet. These surveys are the least expensive, but they have the lowest participation rate and are taken the least seriously.

**6.** Would you prefer reading a printed questionnaire and responding in writing, or would you rather be interviewed by someone? Why?

**7.** Why do telephone interviewers and personal interviewers need to be trained?

Check your answers by turning to the Appendix.

Questionnaires are designed very carefully. Use the questionnaires in the following examples to answer questions 8 and 9.

## Example 1

Maxine's Restaurant conducts a client survey by placing a copy of the following questionnaire at each table.

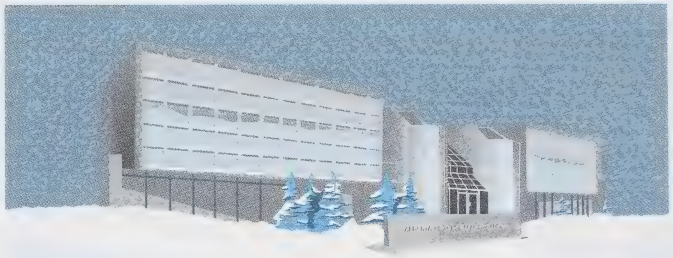
**Customer Survey**

At Maxine's, we welcome your comments. Please fill out this questionnaire and place it in the comments box.

How would you rate these aspects of Maxine's?

| | Excellent | Good | Poor |
|---|---|---|---|
| • employee courtesy | ❑ | ❑ | ❑ |
| • speed of service | ❑ | ❑ | ❑ |
| • food quality | ❑ | ❑ | ❑ |
| • cleanliness | ❑ | ❑ | ❑ |

## Example 2



The Alberta Distance Learning Centre conducts a course survey by placing a questionnaire in the last Student Module Booklet of every course.

Read the Course Survey for Mathematics 7; it can be found in this Student Module Booklet following the Appendix.

**8.** **a.** Why do you think Maxine's Restaurant does not ask for any personal information such as name, address, and age?

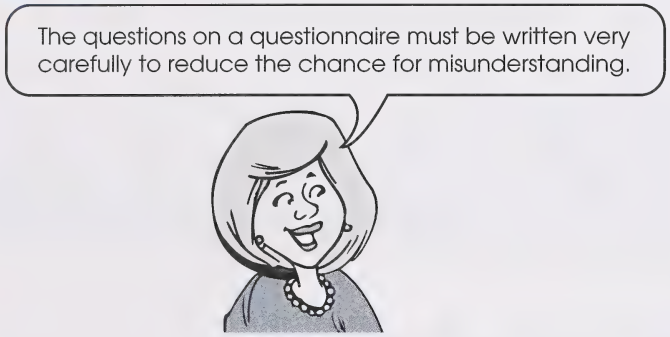
**b.** Why do you think the Alberta Distance Learning Centre requests your name, address, and age?

**9.** Questionnaires may have different styles of questions such as numerical response questions, written response questions, yes/no questions, and multiple-choice questions.

**a.** Name the style(s) of questions used in the questionnaire for Maxine's Restaurant.

**b.** Name the style(s) of questions used in the questionnaire for the Alberta Distance Learning Centre.

Check your answers by turning to the Appendix.




**10.** The following question was asked in the Quebec referendum on October 30, 1995. Do you think this question is well written? Why or why not?

Do you agree that Quebec should become sovereign after having made a formal offer to Canada for a new economic and political partnership, within the scope of the bill respecting the future of Quebec and of the agreement signed on June 12, 1995?

❑ Yes ❑ No

**11.** Choose the question that is more clearly worded in each pair and explain why the other question is unclear.

**a.** A. Are you a member of a large family?

❑ Yes ❑ No

B. How many brothers and sisters do you have?

_____

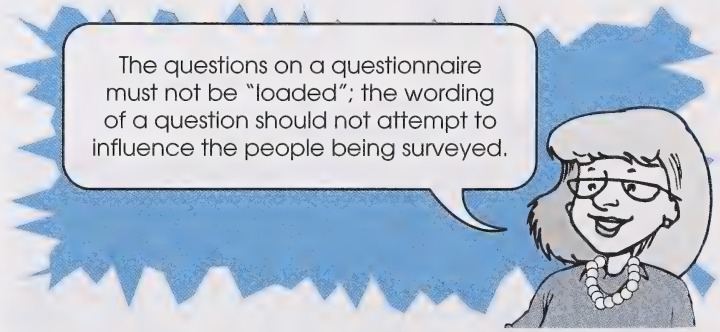
**b.** A. Do you agree that people should not be permitted to obtain a driver's license until they are at least 18 years of age?

❑ Yes ❑ No

B. Do you agree that people should be at least 18 years of age in order to obtain a driver's license?

❑ Yes ❑ No

Check your answers by turning to the Appendix.



**12.** Choose the "unloaded" question in each pair and indicate the bias in the other question.

**a.** A. Do you agree that reckless mountain climbers should pay the expenses of the rescuers who come to their assistance?

❑ Yes ❑ No

B. Do you agree that mountain climbers should pay the expenses of rescuers who come to their assistance?

❑ Yes ❑ No

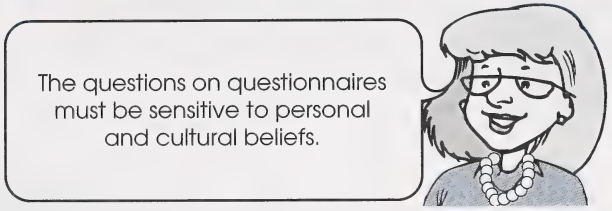
**b.** A. Would you vote for Ms. Jones?

❑ Yes ❑ No

B. Would you vote for that intelligent and hardworking candidate, Ms. Jones?

❑ Yes ❑ No

Check your answers by turning to the Appendix.




Find the article entitled " 'Plaid' Family Would Rather Skip Question" in the Appendix. This article appeared in the *Edmonton Journal* in January 1996. Read the article and then answer the following questions.

**13.** **a.** The 1996 Census asked a question about racial background. Why did Paul and Sita LeGrange Rao consider not answering this question?

**b.** Why do you think Census Canada asked this question?

Check your answers by turning to the Appendix.

## Now Try This


The Angus Reid Group is a Canadian company whose business is doing surveys. If you visit their home page on the Internet, you can view the results of some past surveys. You may even be able to participate in a current poll. This is the uniform resource locator (URL) of the Angus Reid Group's home page.

http://www.Angusreid.com/

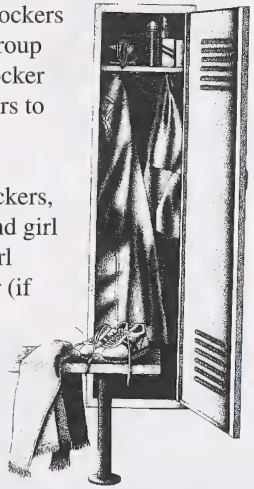
The Gallup Organization is an American company whose business is doing surveys. If you visit their home page on the Internet, you can view the results of some past surveys and discover how their surveys are conducted. You may even be able to participate in a current poll. This is the uniform resource locator (URL) of Gallup's home page.

http://www.gallup.com./

## Now Try This

Use a problem-solving strategy to answer the following question.

**14.** A locker room at a gymnasium has 25 lockers along one wall. An instructor leads a group of 25 girls in a single file through the locker room to the pool and notices all the doors to the lockers are closed.



As the first girl in the line passes the lockers, she opens every locker door. The second girl shuts every second locker. The third girl changes the position of every third door (if the door is open, she closes it; if the door is closed, she opens it). The fourth girl changes the position of every fourth door.

If this pattern continues, how many doors will be closed after the last girl passes by the lockers?

Check your answer by turning to the Appendix.

In this activity you collected data by conducting surveys. You displayed data in a circle graph. You discussed the design of questionnaires and solved a problem involving lockers.

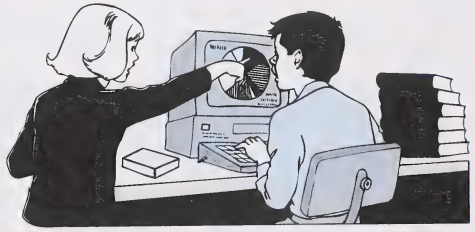
## Follow-up Activities

If you had difficulties understanding the concepts and skills in the activities, it is recommended that you do the Extra Help. If you have a clear understanding of the concepts and skills, it is recommended that you do the Enrichment. You may decide to do both.

## Extra Help

In this section you used a circle graph to report some survey results.

If you had difficulty constructing circle graphs, you may find a computer and a spreadsheet program such as *ClarisWorks*™ very helpful.



## Example

Recall Thach's survey of the fruits in a can of fruit cocktail. Thach recorded his data in a frequency table like this.

| Fruit | Tally | Frequency |
|---|---|---|
| Apples | 𝍸 𝍸 III | 13 |
| Bananas | 𝍸 I | 6 |
| Grapes | II | 2 |
| Peaches | 𝍸 𝍸 I | 11 |
| Pears | IIII | 4 |

Use the given data and a computer spreadsheet program to make a circle graph.

## Solution

**Step 1:** Enter the data in a spreadsheet like this.

File Edit Format Calculate Options View

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Fruit | Frequency | | |
| 2 | Apples | 13 | | |
| 3 | Bananas | 6 | | |
| 4 | Grapes | 2 | | |
| 5 | Peaches | 11 | | |
| 6 | Pears | 4 | | |

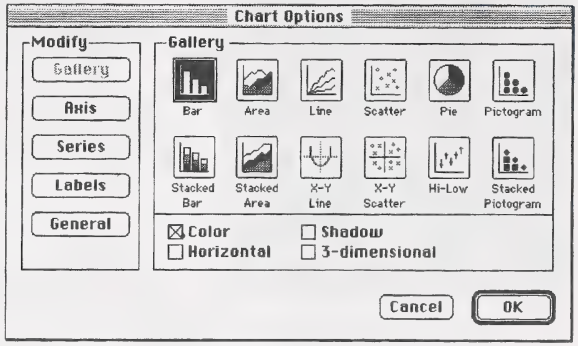
**Step 2:** Select a range of spreadsheet cells. Either click on cell A1 and drag through the cells to cell B6, or click on the first cell (A1) and then shift-click on the last cell (B6). **Note:** The selected cells are highlighted.

File Edit Format Calculate Options View

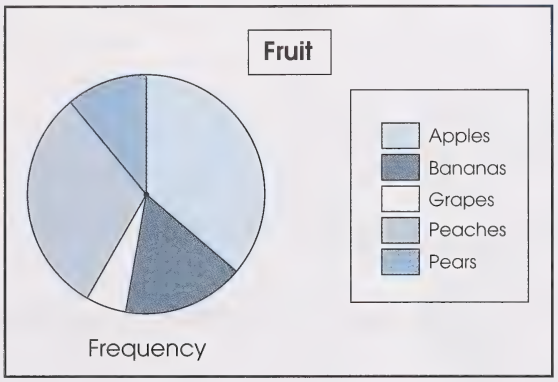
Fruit

| | A | B | C | D |
|---|---|---|---|---|
| 1 | Fruit | Frequency | | |
| 2 | Apples | 13 | | |
| 3 | Bananas | 6 | | |
| 4 | Grapes | 2 | | |
| 5 | Peaches | 11 | | |
| 6 | Pears | 4 | | |

**Step 3:** Click on "Options" in the menu bar. Hold and drag down to "Make Chart." A Chart Options dialogue box like the one shown here will appear.

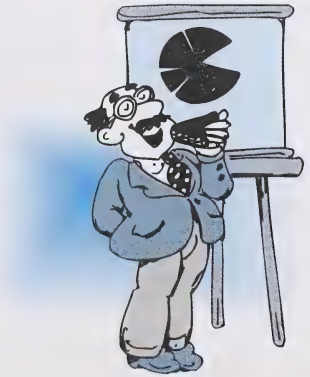


**Step 4:** Click on "Pie" and then either press "Return" or click on the OK button. A pie graph like the one shown here will appear.




**Note:** A circle graph is sometimes called a **pie graph** because it resembles a pie.

You can alter the title of the circle graph by editing the text in cell A1 (see Step 1).



If you have access to a computer and a spreadsheet program such as *ClarisWorks*™, use them to answer questions 1 and 2. If you do not have access to a computer and a spreadsheet program, answer questions 1 and 2 using a compass, protractor, and straightedge.

**1.** Lucien asked 40 adults at random this question:

What is your favourite flower—carnation, daisy, lily, pansy, or rose?

Lucien recorded the responses in a frequency table like this.

| Favourite Flower | Tally | Frequency |
|---|---|---|
| Carnation | 𝍸 𝍸 III | 13 |
| Daisy | 𝍸 IIII | 9 |
| Lily | 𝍸 | 5 |
| Pansy | III | 3 |
| Rose | 𝍸 𝍸 | 10 |

Display this data in a circle graph.

**2.** Samia selected 18 junior high school students at random and asked them this question:

Which of the following pets would you most like to have—bird, cat, dog, fish, or hamster?

Samia recorded the responses in a frequency table like the following.

PHOTO SEARCH LTD.

| Favourite Pet | Tally | Frequency |
|---|---|---|
| Bird | / | 1 |
| Cat | 𝍸 | 5 |
| Dog | 𝍸 / | 6 |
| Fish | //// | 4 |
| Hamster | // | 2 |

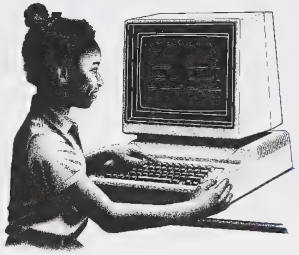
Display this data in a circle graph.

Check your answers by turning to the Appendix.

## Enrichment

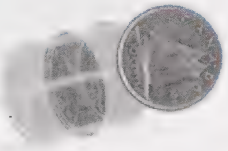
In this section you used techniques such as tossing a coin, twirling a spinner, rolling a cube, or choosing a ball from a bag (without looking) to do probability experiments.

You can also use a computer and a spreadsheet program like *ClarisWorks*™ to simulate probability experiments.



### Example 1

Suppose you toss a coin 20 times. Estimate how many times "Heads" will occur. Explain how you can use a computer to solve this problem.



### Solution

The spreadsheet program in *ClarisWorks*™ has a built-in subprogram designed to call up random numbers. The formula = RAND(2) will randomly select either the number 1 or 2.

Let 1 represent "Heads." Let 2 represent "Tails."

**Step 1:** Make a frequency table like this.

| Event | Tally | Frequency |
|---|---|---|
| Heads | | |
| Tails | | |

**Step 2:** In a spreadsheet, click on cell A1 and enter the formula = RAND(2). Cell A1 will be highlighted and the formula will appear in the entry bar.

File Edit Format Calculate Options View

A1 | × | ✓ | = RAND(2)

| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Step 3:** Press "Return." Either 1 (for "Heads") or 2 (for "Tails") will appear in cell A1.

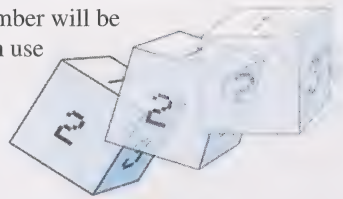
**Step 4:** Record the result of the coin toss simulation in the frequency table.

**Step 5:** To obtain another random number, click again on cell A1 in the spreadsheet. Then click on "Calculate" in the menu bar. Continue holding and drag down to "Calculate Now." Another randomly selected whole number (either 1 or 2) will appear in cell A1.

**Step 6:** Repeat steps 4 and 5 until 20 coin tosses have been simulated.

## Example 2

You have a cube with faces labelled 1, 2, 3, 4, 5, and 6. Suppose you rolled the cube 30 times. Estimate how often each number will be rolled. Explain how you can use a computer to solve this problem.



## Solution

The spreadsheet program in *ClarisWorks*™ has a built-in subprogram designed to call up random numbers. The formula = RAND(6) will randomly select the number 1, 2, 3, 4, 5, or 6.

**Step 1:** Make a frequency table like this.

| Event | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

**Step 2:** In a spreadsheet, click on cell A1 and enter the formula = RAND(6), but do not press "Return" yet. Cell A1 will be highlighted, and the formula = RAND(6) will appear in the entry bar.

File Edit Format Calculate Options View

A1 × √ = RAND(6)

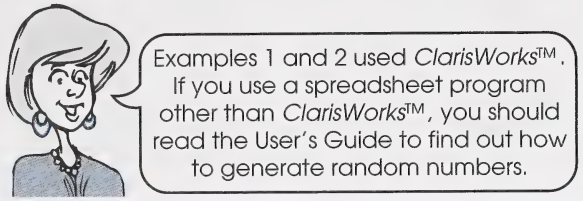
| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |

**Step 3:** Press "Return". A randomly selected whole number from 1 to 6 will appear in cell A1.

**Step 4:** Record the result of the cube roll simulation in the frequency table.

**Step 5:** To obtain another random number, click again on cell A1 in the spreadsheet. Then click on "Calculate" in the menu bar and drag down to "Calculate Now." Another randomly selected number from 1 and 6 will appear in cell A1.

**Step 6:** Repeat steps 4 and 5 until 30 rolls of the cube have been simulated.




If you have access to a computer and a spreadsheet program such as *ClarisWorks*™, use them to answer questions 1 and 2. If you do not have access to a computer and a spreadsheet program, use tiles of paper and a bag to answer questions 1 and 2.

**1.** Suppose you want to randomly select a group of people so that someone in the group has a birthday each month. Estimate how many people you should pick. **Hint:** There are 12 months in a year.

**a.** Give the steps you would use to solve this problem.
**b.** Solve the problem.

**2.** Suppose you want to randomly select a group of people so that two people in the group share the same birthday (month and day). Estimate how many people you should pick. **Hint:** There are 366 days in a leap year.



**a.** Give the steps you would use to solve this problem.
**b.** Solve the problem.

Check your answers by turning to the Appendix.

## Conclusion


WESTFILE INC.

In this section you collected first-hand data by doing probability experiments and conducting surveys. You used the probability formula, experiments, statistics, and the Monte-Carlo method to make predictions. You also used surveys to describe a group of people, animals, or things.

You could conduct a survey to discover if potato chips, chocolate bars, popcorn, ice cream, or fruit is the favourite snack food of teenagers. You could also find out which flavour of potato chips or ice cream is most popular. As well, you could find out how many snack foods are eaten in a week. What other questions could you research?

## Assignment

Assignment Booklet

You are now ready to complete the module assignment for Section 2.

# Module Summary

JIM WHITMER PHOTOGRAPHY

In this module you increased your skills in reading and interpreting second-hand data, gathering and reporting your own data, and making predictions.

You are living in the information age. More than ever before, governmental agencies, businesses, and individuals are concerned with using numerical information to make decisions.

Governments use data on hospital admission levels to decide on the need for new hospitals. Insurance companies use data on motor vehicle accidents and traffic violations to decide insurance rates. Baseball coaches use team statistics to decide on game strategies. You use weather forecasts when planning a day's activities.

## Final Module Assignment

Assignment Booklet

You are now ready to complete the final module assignment.

You are also ready to complete the Course Survey for Mathematics 7. As mentioned previously, it is in this Student Module Booklet following the Appendix.

# Course Summary

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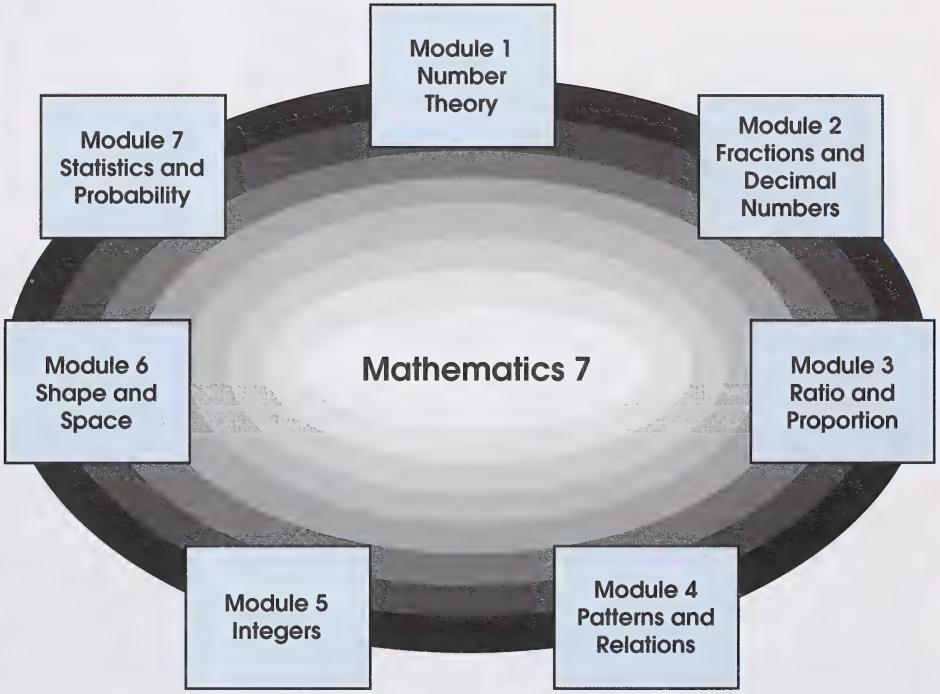



Congratulations on completing the seven modules of Mathematics 7. We hope you found the course interesting, fun, and challenging.

By now all your assignments should have been completed and reviewed. Now it's time for the last step in the Mathematics 7 course—the Final Test. With all the hard work and time you put into the course, we hope you do well!

# APPENDIX

- Glossary
- Suggested Answers
- Newspaper Articles

# Glossary

**Clusters:** values grouped closely together in a set of data

**Data:** another word for statistics

**Database:** collection of data organized for rapid research and retrieval; also called **data bank**

**Equally likely:** having the same chance of occurring

**Favourable outcomes:** results that you want to happen

**Frequency:** the number of times a particular action or event has occurred

**Frequency table:** a chart that lists a set of events together with the number of occurrences of each event

**Gaps:** spaces between values in a set of data

**Independent events:** events that do not depend on each other, for example, twirling a spinner and tossing a coin

**Interquartile range:** the difference between the upper quartile and the lower quartile

**Line plot:** a simple way of organizing and displaying a set of data

**Lower extreme:** the least value in a set of data

**Lower quartile:** the median of the values less than the median in a set of data

**Mean:** the arithmetic average

**Median:** the middle value or the average of the two middle values in a set of data arranged in ascending order

**Mode:** the most frequently occurring value(s) in a set of data

**Monto-Carlo method:** a technique where an event is modelled using probability tools such as coins, cubes, spinners, and balls; also called a **simulation**

**Outcome:** the result of an action, such as flipping a coin

**Poll:** an opinion survey

**Population:** the set of people, animals, or things being studied

**Possible outcomes:** results that could happen

**Probability:** a number from 0 to 1 that tells how likely an event is to happen

**Probability theorem:** the ratio of the number of favourable outcomes (what you want to happen) to the number of possible outcomes (what could happen)

**Problem:** a task for which the method of finding the answer (as well as the answer) is not immediately known

**Random:** occurring by chance; not influenced by outside factors

**Range:** the difference between the upper extreme and the lower extreme in a set of data

**Relative frequency:** the frequency of a given outcome compared to the total number of trials; also called the **experimental probability** of an event

**Sample:** a part of the population used to obtain information about the whole group

**Statistical survey:** a collection of data that describes some aspect of a set of people, animals, or things

**Statistics:** items of information which have been collected and recorded; the branch of mathematics that deals with the systematic collection and organization of numerical information

**Stem and leaf plot:** a way of organizing and displaying a set of data in a T-table

**Upper extreme:** the greatest value in a set of data

**Upper quartile:** the median of the values greater than the median in a set of data

# Suggested Answers

## Section 1: Activity 1

**1.** Answers may vary depending on the year the topic was researched. Different sources may also have varying information.

**a.** The greatest distance a baby carriage (pram) has been pushed in 24 h is 563.62 km. (This record was set in Belgium in 1988.) Source: *The Guinness Book of Records*

**b.** The following chart lists the ten largest freshwater lakes in the world in descending order. Source: encyclopedias such as *The Junior Encyclopedia of Canada*

**The World's Ten Largest Lakes***

| Lake | Location | Area ($km^2$) |
|---|---|---|
| Superior | Canada/United States | 84 243 |
| Victoria | Africa | 69 748 |
| Huron | Canada/United States | 63 096 |
| Michigan | United States | 57 757 |
| Tanganyika | Africa | 33 020 |
| Baikal | Russia | 31 621 |
| Great Bear | Canada | 31 326 |
| Malawi | Africa | 28 990 |
| Great Slave | Canada | 28 568 |
| Erie | Canada/United States | 25 812 |

*Not including saltwater lakes, such as Caspian Sea (372 400 $km^2$) and Aral Sea (64 700 $km^2$)

**c.** Canadian Elvis Stojko was the Men's World Figure Skating champion in 1995. Source: almanacs such as *The Sports Almanac* and *The World Almanac and Book of Facts*, magazines, newspapers, the Internet

**d.** The government expenditures on culture by each province or territory for the 1990–91 fiscal year are listed in the following chart. Source: *The Canadian World Almanac and Book of Facts*

| Government Spending on Culture 1990–1991 | |
|---|---|
| **Province or Territory** | **Amount ($)** |
| Newfoundland | 23 529 000 |
| Prince Edward Island | 10 760 000 |
| Nova Scotia | 60 444 000 |
| New Brunswick | 27 631 000 |
| Quebec | 536 387 000 |
| Ontario | 597 833 000 |
| Manitoba | 79 198 000 |
| Saskatchewan | 62 385 000 |
| Alberta | 160 373 000 |
| British Columbia | 209 463 000 |
| Yukon | 7 744 000 |
| Northwest Territories | 12 700 000 |

SOURCE OF DATA: STATISTICS CANADA

**e.** In 1990, the average number of hours per week Canadians spent watching television was 23.3 h. Source: *The Canadian World Almanac and Book of Facts*

**f.** The busiest airport in the world is located in Chicago, U.S.A. (In 1992, there were over 64 million passenger arrivals or departures from this airport.) Source: *The World Almanac and Book of Facts*

**g.** In 1994, the population of Ottawa-Hull was about 980 500. Source: the Internet, an encyclopedia, an atlas, *The Canadian World Almanac and Book of Facts*

**h.** By the end of 1992, the motion picture (movie) that had made the most money was *E.T. The Extra-Terrestrial*. Source: *The World Almanac and Book of Facts*

**2.** The population of Ottawa-Hull will change over time. The list of the ten largest lakes may be different depending on who measured the lakes and how accurate the measurements were.

**3. a.** The age intervals are under 15, 15–19, 20–24, 25–29, 30–34, 35–39, 40–44, and 45 and over.

**b.** The interval with the highest number of first-borns in 1931, 1950, and 1971 was 20–24. In 1990, it was 25–29.

**c.** Answers may vary. Newfoundland may not keep data on the age of the mother at the birth of her first child.

**4. a.** No, the name *Linda* did not rank in the top 15 in 1970 or 1990.

**b.** Yes, in 1970, the name *Robert* ranked third in popularity, and in 1990, the name *Robert* ranked fifteenth.

**c.** The name *David* has continued to rank in the top 15 names. (In 1950, the name *David* ranked second; in 1970, it ranked second; and in 1990, it ranked eleventh.)

The name *Michael* has continued to rank in the top 15 names. (In 1950, the name *Michael* ranked eleventh; in 1970, it ranked first; and in 1990, it ranked first.)

The name *James* has continued to rank in the top 15 names. (The name *James* ranked fourth in 1950, fifth in 1970, and thirteenth in 1990.)

**5. a.** In 1990, there were 98 771 000 000 $m^3$ of natural gas produced in Canada. This had a value of $5 692 000 000.

**b.** In 1990, there was no marketable natural gas produced in New Brunswick. There was no value.

**c.** In 1970, there were 2 000 000 $m^3$ of natural gas produced in the Northwest Territories. This had a value too small to be included in the data.

**d.** In 1990, Alberta was the leading producer.

**e.** From 1980 to 1990, the production of natural gas decreased in New Brunswick and the Northwest Territories.

**f.** From 1980 to 1990, the production of natural gas increased in Ontario, Saskatchewan, Alberta, and British Columbia.

**6. a.** The data are arranged alphabetically.

**b.** From 1960 to 1990, the Montreal Canadiens won the Stanley Cup the most times.

**c.** The Edmonton Oilers won the Stanley Cup 5 times in this time period.

**d.** The Calgary Flames won the Stanley Cup once in this time period.

**7. a.** The graph compares the number of passengers at Canada's five busiest airports in 1985.

**b.** The data are arranged in order of rank from greatest to least.

**c.**  represents 75 000 passengers,  represents 50 000 passengers, and  represents 25 000 passengers.

**d.** About 1 500 000 passengers used Toronto's airport.

**e.** About 725 000 passengers used Vancouver's airport.

**f.** About 1 300 000 more passengers used Toronto's airport than Winnipeg's airport.

**8. a.** The Olympic ice hockey medal winners from 1924 to 1984 are being compared.

**b.** The data are arranged in alphabetical order.

**c.** Canada won the most medals during this period of time.

**d.** Canada, Great Britain, the Soviet Union, and the United States won gold medals during this period of time.

**e.** The Soviet Union won the most gold medals during this period of time.

**f.** Canada won 5 gold medals during this period of time.

**g.** Canada won 2 silver medals during this period of time.

**h.** Canada won 2 bronze medals during this period of time.

**9. a.** The average single-family housing prices in selected Canadian cities in 1981 and 1987 are being compared.

**b.** The data are arranged alphabetically.

**c.** In 1981, houses cost the most in Vancouver. In 1987, houses cost the most in Toronto.

**d.** During this time period, house prices increased in Regina, Toronto, and Winnipeg. Toronto showed the largest increase.

**e.** During this time period, house prices decreased in Edmonton and Vancouver. Edmonton showed the largest decrease.

**10. a.** The percentages of illiterate adults in the developing world in 1990 are being compared.

**b.** In 1990, South Asia had the greatest percentage of illiteracy for men. South Asia had the greatest percentage of illiteracy for women.

**c.** In 1990, Latin America had the least percentage of illiteracy for men. Latin America had the least percentage of illiteracy for women.

**d.** There was a higher percentage of illiterate women than men in these areas of the developing world.

**e.** South Asia had the greatest difference in the percentage of men and women who were illiterate.

**f.** Latin America had the least difference in the percentage of men and women who were illiterate.

**11. a.** The distribution of the land surface of Earth is being shown in the circle graph.

**b.** Asia has the greatest land surface.

**c.** North America ranks third in the amount of land area, after Asia and Africa.

**d.** North America has 16% of Earth's land surface.

$$\frac{16}{100} \overset{\times 1\,554\,000}{=} \frac{24\,864\,000}{155\,400\,000}$$

There are about 24 864 000 $km^2$ of land in North America.

**12. a.** The amount of monthly precipitation in Edmonton in 1995 is being compared in the histogram.

**b.** August had the most precipitation.

**c.** There were 17.6 mm of precipitation in Edmonton in April, 1995. Of this precipitation, 11.6 mm was rainfall and 6 mm was snowfall (in rainfall equivalent).

**13. a.** The population distribution by age in Canada in 1991 is shown in the histogram.

**b.** The data are grouped into five age classes: 0–14 years, 15–29 years, 30–44 years, 45–59 years, and 60+.

**c.** In 1991, the age groups in order from greatest to least percent of the population were 30–44 years, 15–29 years, 0–14 years, 60+, and 45–59 years.

**14. a.** The population of Canada from 1901 to 1991 is shown in the broken-line graph.

**b.** The data are arranged chronologically (in order of time).

**c.** The population of Canada was about 7 million in 1911, 11 million in 1931, 14 million in 1951, 22 million in 1971, and 27 million in 1991.

**d.** The population of Canada is increasing.

**15. a.** In 1967, about 78% of men (15 years and over) were participating in the labour force. In 1991, about 75% of men (15 years and over) were participating in the labour force.

**b.** The percentage of men (15 years and over) who participate in the labour force has decreased slightly from 1967 to 1991.

**c.** The percentage will probably be about the same; so, about 75% of the men (15 years and over) will be participating in the labour force in 30 years time.

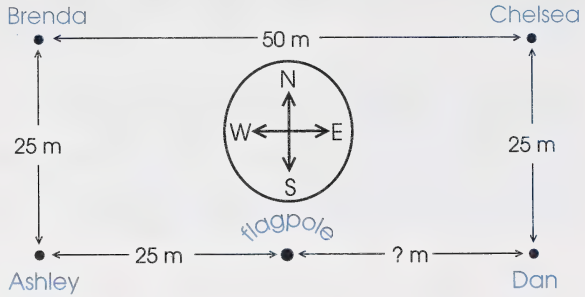
**d.** In 1967, about 35% of women (15 years and over) were participating in the labour force. In 1991, about 59% of women (15 years and over) were participating in the labour force.

**e.** The percentage of women (15 years and over) who participate in the labour force has increased.

**f.** The percentage will probably be higher. It may be as high as 75%, the percentage predicted for men (15 years and over).

## Now Try This

**16.** Make a diagram to help you solve this problem.




Dan was 25 m from the flagpole.

## Section 1: Activity 2

**1. a.** Jean allowed 3 goals in one game.
**b.** Jean allowed fewer than 3 goals in three games.
**c.** Jean allowed more than 3 goals in three games.

**2. a.** **Step 1:** Find the total score.

$$80 + 96 + 84 + 92 + 90 = 442$$

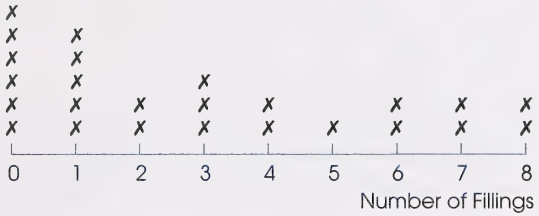
**Step 2:** Find the mean by dividing the total score by the number of games.

$$\frac{442}{5} = 88.4$$

Sally's mean score was 88.4.

**b.** Sally made the mean score in no games.
**c.** Sally made more than the mean score in three games.
**d.** Sally made less than the mean score in two games.

**3. Step 1:** Make a line plot.




**Step 2:** Use the line plot to find the total number of fillings.

$$\begin{aligned}&(6\times0)+(5\times1)+(2\times2)+(3\times3)+(2\times4)\\&\quad+(1\times5)+(2\times6)+(2\times7)+(2\times8)\\&=0+5+4+9+8+5+12+14+16\\&=73\end{aligned}$$

**Step 3:** Find the mean. Divide the total number of fillings by the number of students.

$$\begin{aligned}\frac{73}{25}&=2.92\\&\doteq 3\end{aligned}$$

The mean number of fillings was about 3.

**4.** Arrange the data in ascending order. Because there is an odd number of values, circle the middle value.

| | | | | |
|---|---|---|---|---|
| 16 | 16 | 17 | 18 | (18) |
| 18 | 19 | 19 | 19 | |

There are four values less than the median and four values greater than the median.

The median number of petals per daisy is 18.

**5.** Arrange the data in ascending order. Because there is an even number of values, put a line between the middle two values. Then find the mean of the middle two values.

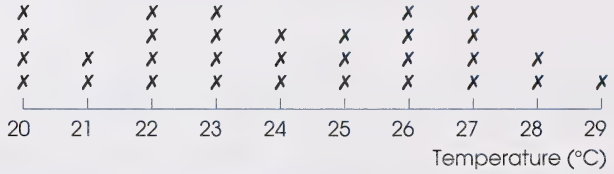
| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 3 | 4 \| |
| 5 | 6 | 6 | 7 | 8 |

There are five values less than the median and five values greater than the median.

$$\begin{aligned}\frac{4+5}{2}&=\frac{9}{2}\\&=4.5\end{aligned}$$

The median number of attempts was 4.5.

**6. Step 1:** Record the data in a line plot.



**Step 2:** Use the line plot to arrange the data in ascending order. Because there is an odd number of data, circle the middle value.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | 20 | 20 | 20 | 21 | 21 | 22 | 22 |
| 22 | 22 | 23 | 23 | 23 | 23 | 24 | (24) |
| 24 | 25 | 25 | 25 | 26 | 26 | 26 | 26 |
| 27 | 27 | 27 | 27 | 28 | 28 | 29 | |

The median high temperature was 24°C.

**7. Step 1:** Make a stem and leaf plot. You may arrange the data in intervals of 10 or intervals of 5. The following answer shows intervals of 10.

| Stem | Leaf |
|---|---|
| 3 | 8 |
| 4 | 8, 3 |
| 5 | 1, 4, 3, 3, 9, 4, 7, 6 |
| 6 | 8, 4, 8, 8 |
| 7 | 7, 9, 2, 7, 3, 9, 6 |
| 8 | 0, 2, 2, 9, 2, 6, 1, 5 |

**Step 2:** Order the numbers in each leaf from least to greatest.

| Stem | Leaf |
|---|---|
| 3 | 8 |
| 4 | 3, 8 |
| 5 | 1, 3, 3, 4, 4, 6, 7, 9 |
| 6 | 4, 8, 8, 8 |
| 7 | 2, 3, 6, 7, 7, 9, 9 |
| 8 | 0, 1, 2, 2, 2, 5, 6, 9 |

**Step 3:** Use the stem and leaf plot to arrange the values in ascending order.

Because there is an even number of values, put a line between the middle two values. Then find the mean of the middle two values.

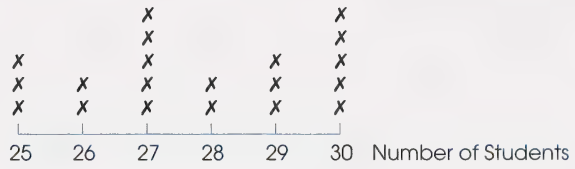
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 38 | 43 | 48 | 51 | 53 | 53 | 54 | 54 |
| 56 | 57 | 59 | 64 | 68 | 68 | 68 | 72 |
| 73 | 76 | 77 | 77 | 79 | 79 | 80 | 81 |
| 82 | 82 | 82 | 85 | 86 | 89 | | |

The median is between 68 and 72.

$$\frac{68+72}{2} = \frac{140}{2}$$
$$= 70$$

The median score on the English assignment was 70.

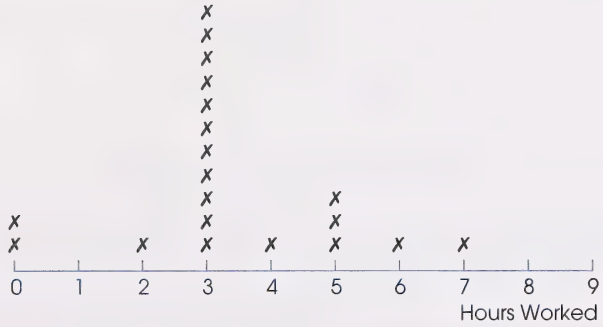
**8.** Make a line plot. Then find the value(s) that occurred most frequently.




The modes are 27 and 30 students per class.

**Note:** Because there are two modes, the data are called bimodal.

**9.** Make a line plot. Then find the value(s) that occurred most frequently.




The mode is 3 h.

## Now Try This

**10.** You can use logical reasoning and elimination to solve this problem.

| | Art | Bill | Clive |
|---|---|---|---|
| Red | ✗ | ✓ | ✗ |
| Blue | ✗ | ✗ | ✓ |
| Green | ✓ | ✗ | ✗ |

Art chose green, Bill chose red, and Clive chose blue.

# Section 1: Activity 3

**1. a.** The lower extreme is the year 1867.
**b.** The upper extreme is the year 1949.

**c.** Subtract the lower extreme from the upper extreme.

$$1949 - 1867 = 82$$

The range is 82 years.

**2. a.** The lower extreme is 71 940 visitors.

**b.** The upper extreme is 296 330 visitors.

**c.** Subtract the lower extreme from the upper extreme.

$$296\,330 - 71\,940 = 224\,390$$

The range is 224 390 visitors.

**3. a.** The lower extreme is a population of 28 000.

**b.** The upper extreme is a population of 10 085 000.

**c.** Subtract the lower extreme from the upper extreme.

$$10\,085\,000 - 28\,000 = 10\,057\,000$$

The range is 10 057 000 people.

**4. a.** List the data is ascending order and find the median.

0 1 1 2 3 3 3 | 3 4 4 4 5 5 6

$$\frac{3+3}{2} = \frac{6}{2}$$
$$= 3$$

The median is 3 goals.

**b.** Find the median of the values less than the median.

0 1 1 (2) 3 3 3 | 3 4 4 4 5 5 6

The lower quartile is 2 goals.

**c.** Find the median of the values greater than the median.

0 1 1 (2) 3 3 3 | 3 4 4 (4) 5 5 6

The upper quartile is 4 goals.

**d.** Subtract the lower quartile from the upper quartile.

$$4 - 2 = 2$$

The interquartile range is 2 goals.

**5. a.** List the data in ascending order and find the median.

0 0 1 1 (2) 8 15 16 18

The median number of satellites is 2.

**b.** Find the median of the values less than the median.

0 0 | 1 1 (2) 8 15 16 18

$$\frac{0+1}{2} = \frac{1}{2}$$
$$= 0.5$$

The lower quartile of this set of data is 0.5 satellites.

**c.** Find the median of the values greater than the median.

0 0 | 1 1 (2) 8 15 | 16 18

$$\frac{15+16}{2} = \frac{31}{2}$$
$$= 15.5$$

The upper quartile is 15.5 satellites.

**d.** Subtract the lower quartile from the upper quartile.

$$15.5 - 0.5 = 15$$

The interquartile range is 15 satellites.

**6. a. Step 1:** Make a stem and leaf plot. You may arrange the data in intervals of 10 or intervals of 5. The following answer shows intervals of 10.

| Stem | Leaf |
|---|---|
| 3 | 8 |
| 4 | |
| 5 | 8 |
| 6 | 8, 1, 2, 3, 8, 3, 0, 9 |
| 7 | 3, 7, 1, 2, 2, 8, 8, 9, 4 |
| 8 | 3, 1, 2, 4, 3, 0, 3 |
| 9 | 6, 5, 9 |

**Step 2:** Order the numbers in each leaf from least to greatest.

| Stem | Leaf |
|---|---|
| 3 | 8 |
| 4 | |
| 5 | 8 |
| 6 | 0, 1, 2, 3, 3, 8, 8, 9 |
| 7 | 1, 2, 2, 3, 4, 7, 8, 8, 9 |
| 8 | 0, 1, 2, 3, 3, 3, 4 |
| 9 | 5, 6, 9 |

**Step 3:** Use the stem and leaf plot in Step 2 to arrange the values in ascending order. Then find the median.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 38 | 58 | 60 | 61 | 62 | 63 | 63 | 68 |
| 68 | 69 | 71 | 72 | 72 | 73 | (74) | 77 |
| 78 | 78 | 79 | 80 | 81 | 82 | 83 | 83 |
| 83 | 84 | 95 | 96 | 99 | | | |

The median is a score of 74.

**Step 4:** To determine the lower quartile, find the median of the values less than the median.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 38 | 58 | 60 | 61 | 62 | 63 | 63 | 68 |
| 68 | 69 | 71 | 72 | 72 | 73 | (74) | 77 |
| 78 | 78 | 79 | 80 | 81 | 82 | 83 | 83 |
| 83 | 84 | 95 | 96 | 99 | | | |

The lower quartile is between 63 and 68.

$$\frac{63+68}{2} = \frac{131}{2}$$
$$= 65.5$$

The lower quartile is a score of 65.5.

**Step 5:** To determine the upper quartile, find the median of the values greater than the median.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 38 | 58 | 60 | 61 | 62 | 63 | 63 | 68 |
| 68 | 69 | 71 | 72 | 72 | 73 | (74) | 77 |
| 78 | 78 | 79 | 80 | 81 | 82 | 83 | 83 |
| 83 | 84 | 95 | 96 | 99 | | | |

The upper quartile is between 82 and 83.

$$\frac{82+83}{2} = \frac{165}{2}$$
$$= 82.5$$

The upper quartile is a score of 82.5.

**Step 6:** Find the interquartile range by subtracting the lower quartile from the upper quartile.

$$82.5 - 65.5 = 17$$

The interquartile range is 17 marks.

**b.** Find the range by subtracting the lower extreme from the upper extreme.

$$99 - 38 = 61$$

The range is 61 years.

**7. a. Step 1:** Make a stem and leaf plot. You may arrange the data in intervals of 10 or intervals of 5. The following answer shows intervals of 10.

| Stem | Leaf |
|---|---|
| 1 | 5, 7, 5, 2, 1, 8, 0 |
| 2 | 3, 8, 8, 3, 7, 0, 4 |
| 3 | 7, 9, 6, 8, 5, 2, 5, 2 |
| 4 | 5, 3, 4, 2, 0 |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | 5, 3, 0 |

**Step 2:** Order the numbers in each leaf from least to greatest.

| Stem | Leaf |
|---|---|
| 1 | 0, 1, 2, 5, 5, 7, 8 |
| 2 | 0, 3, 3, 4, 7, 8, 8 |
| 3 | 2, 2, 5, 5, 6, 7, 8, 9 |
| 4 | 0, 2, 3, 4, 5 |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | 0, 3, 5 |

**Step 3:** Use the stem and leaf plot in Step 2 to arrange the values in ascending order. Then find the median.

| | | | | | |
|---|---|---|---|---|---|
| 10 | 11 | 12 | 15 | 15 | 17 |
| 18 | 20 | 23 | 23 | 24 | 27 |
| 28 | 28 | 32 \| | 32 | 35 | 35 |
| 36 | 37 | 38 | 39 | 40 | 42 |
| 43 | 44 | 45 | 100 | 103 | 105 |

The median is between 32 and 32.

$$\frac{32+32}{2} = \frac{64}{2}$$
$$= 32$$

The median is \$32.

**Step 4:** To determine the lower quartile, find the median of the values less than the median.

| | | | | | |
|---|---|---|---|---|---|
| 10 | 11 | 12 | 15 | 15 | 17 |
| 18 | (20) | 23 | 23 | 24 | 27 |
| 28 | 28 | 32 \| | 32 | 35 | 35 |
| 36 | 37 | 38 | 39 | 40 | 42 |
| 43 | 44 | 45 | 100 | 103 | 105 |

The lower quartile is \$20.

**Step 5:** To determine the upper quartile, find the median of the values greater than the median.

| | | | | | |
|---|---|---|---|---|---|
| 10 | 11 | 12 | 15 | 15 | 17 |
| 18 | (20) | 23 | 23 | 24 | 27 |
| 28 | 28 | 32 \| | 32 | 35 | 35 |
| 36 | 37 | 38 | 39 | (40) | 42 |
| 43 | 44 | 45 | 100 | 103 | 105 |

The upper quartile is \$40.

**Step 6:** Find the interquartile range by subtracting the lower quartile from the upper quartile.

$$40 - 20 = 20$$

The interquartile range is \$20.

**b.** Find the range by subtracting the lower extreme from the upper extreme.

$$105 - 10 = 95$$

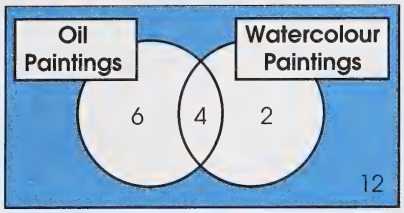
The range is \$95.

**8.** There are gaps. There are no scores below 38 and there are no scores in the 40s. There are clusters. Most of the scores are in the 60s, 70s, and 80s.

**9.** There are gaps. There are no savings in the 50s, 60s, 70s, 80s, or 90s. There are clusters. Most of the savings are in the 10s, 20s, 30s, and 40s.

## Now Try This

**10.** You can use a Venn diagram to solve this problem.




Twelve students do not like to do either oil or watercolour paintings.

# Section 1: Follow-up Activities

## Extra Help

**1. Step 1:** Make a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 1 | 6 |
| 2 | 9, 9, 7, 5, 9 |
| 3 | 4, 6, 8 |
| 4 | 9, 4, 6 |
| 5 | 0, 0 |
| 6 | 4 |

**Step 2:** Rearrange the numbers in each leaf from least to greatest.

| Stem | Leaf |
|---|---|
| 1 | 6 |
| 2 | 5, 7, 7, 9, 9, 9 |
| 3 | 4, 6, 8 |
| 4 | 4, 6, 9 |
| 5 | 0, 0 |
| 6 | 4 |

**Step 3:** Use the stem and leaf plot in Step 2 to arrange the data in ascending order.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 25 | 27 | 27 | 29 | 29 | 29 | 34 |
| 36 | 38 | 44 | 46 | 49 | 50 | 50 | 64 |

**a.** The lower extreme is 16 grams of fat.

**b.** The upper extreme is 64 grams of fat.

**c.** Subtract the lower extreme from the upper extreme.

$64 - 16 = 48$

The range is 48 grams of fat.

**d.** **Step 1:** Find the total number of grams of fat.

$$16+25+(2\times27)+(3\times29)+34+36+38$$
$$+44+46+49+(2\times50)+64$$
$$=593$$

**Step 2:** Divide the total number of grams of fat by the number of entrees.

$$\frac{593}{16}\doteq 37$$

The mean is about 37 grams of fat.

**e.** The mode is 29 grams of fat.

**f.** Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 25 | 27 | 27 | 29 | 29 | 29 | 34 \| |
| 36 | 38 | 44 | 46 | 49 | 50 | 50 | 64 |

$$\frac{34+36}{2}=\frac{70}{2}$$
$$=35$$

The median is 35 grams of fat.

**g.** Consider only the values less than the median. Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 25 | 27 | 27 \| | 29 | 29 | 29 | 34 \| |
| 36 | 38 | 44 | 46 | 49 | 50 | 50 | 64 |

$$\frac{27+29}{2}=\frac{56}{2}$$
$$=28$$

The lower quartile is 28 grams of fat.

**h.** Consider only the values greater than the median. Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 16 | 25 | 27 | 27 \| | 29 | 29 | 29 | 34 \| |
| 36 | 38 | 44 | 46 | 49 | 50 | 50 | 64 |

$$\frac{46+49}{2}=\frac{95}{2}$$
$$=47.5$$

The upper quartile is 47.5 grams of fat.

**i.** Subtract the lower quartile from the upper quartile.

$$47.5-28=19.5$$

The interquartile range is 19.5 grams of fat.

**2.** Examine the stem and leaf plot in Step 2 of question 1.

| Stem | Leaf |
|---|---|
| 1 | 6 |
| 2 | 5, 7, 7, 9, 9, 9 |
| 3 | 4, 6, 8 |
| 4 | 4, 6, 9 |
| 5 | 0, 0 |
| 6 | 4 |

There are no gaps. There are clusters. Most of the data is in the 20s, 30s, and 40s.

**3.** You may use a spreadsheet program to sort the data in ascending order.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 31 | 36 | 40 | 41 | 43 | 47 |
| 48 | 51 | 52 | 53 | 56 | 59 | 61 | 65 |
| 65 | 65 | 67 | 71 | 78 | 83 | 100 | 108 |

**a.** The lower extreme is 3 days of snowfall.

**b.** The upper extreme is 108 days of snowfall.

**c.** Subtract the lower extreme from the upper extreme.

$$108 - 3 = 105$$

The range is 105 days of snowfall.

**d.** **Step 1:** Find the total number of days.

$$\begin{aligned} &3+6+31+36+40+41+43+47+48 \\ &\quad +51+52+53+56+59+61+(6\times 65) \\ &\quad +67+71+78+83+100+108 \\ &=1329 \end{aligned}$$

**Step 2:** Divide the total number of days by the number of cities.

$$\frac{1329}{24} \doteq 55$$

The mean number of days of snowfall is about 55.

**e.** The mode is 65 days of snowfall.

**f.** Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 31 | 36 | 40 | 41 | 43 | 47 |
| 48 | 51 | 52 | 53 | 56 | 59 | 61 | 65 |
| 65 | 65 | 67 | 71 | 78 | 83 | 100 | 108 |

$$\begin{aligned} \frac{53+56}{2} &= \frac{109}{2} \\ &= 54.5 \end{aligned}$$

The median is 54.5 days of snowfall.

**g.** Consider only the values less than the median. Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 31 | 36 | 40 | 41 | 43 | 47 |
| 48 | 51 | 52 | 53 | 56 | 59 | 61 | 65 |
| 65 | 65 | 67 | 71 | 78 | 83 | 100 | 108 |

$$\frac{41+43}{2} = \frac{84}{2}$$
$$= 42$$

The lower quartile is 42 days of snowfall.

**h.** Consider only the values greater than the median. Because there is an even number of values, put a line between the middle two values. Then find the mean of these values.

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 6 | 31 | 36 | 40 | 41 | 43 | 47 |
| 48 | 51 | 52 | 53 | 56 | 59 | 61 | 65 |
| 65 | 65 | 67 | 71 | 78 | 83 | 100 | 108 |

$$\frac{65+67}{2} = \frac{132}{2}$$
$$= 66$$

The upper quartile is 66 days of snowfall.

**i.** Subtract the lower quartile from the upper quartile.

$$66 - 42 = 24$$

The interquartile range is 24 days of snowfall.

**4.** Use the ordered list you made in question 3 and arrange the data in a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 0 | 3, 6 |
| 1 | |
| 2 | |
| 3 | 1, 6 |
| 4 | 0, 1, 3, 7, 8 |
| 5 | 1, 2, 3, 6, 9 |
| 6 | 1, 5, 5, 5, 7 |
| 7 | 1, 8 |
| 8 | 3 |
| 9 | |
| 10 | 0, 8 |

There are gaps. There are no values in the 10s, in the 20s, and in the 90s.

There are clusters. Most of the values are in the 40s, 50s, and 60s.

## Enrichment

**1. a.** A broken-line graph is pictured.

**b.** The vertical axis represents the profits and the horizontal line represents the time.

**c.** The profits went down. Answers will vary as to "why." One reason may be that the boss was on vacation and the employees didn't work as hard.

**2. a.** The driver left home at 08:00.

**b.** The driver ate lunch from 12:00 to 13:00.

**c.** The driver arrived home at 17:00.

**d.** The driver may be a travelling salesperson, taxi driver, delivery person, etc. The only times of the day that gasoline wasn't consumed was during early morning, lunch, and the supper hour.

**e.** The driver purchased gas at 13:00.

**f.** The capacity of the gas tank is 50 L.

**g.** The driver purchased 35 L of gasoline $(50 - 15 = 35)$.

**h.** At 18:00, there was 10 L of gasoline in the gas tank.

**3. a.** From 1981 to 1985, the number of space shuttle launches increased.

**b.** There were two launches in 1986, none in 1987, and two launches in 1988.

**c.** The Challenger space shuttle exploded in 1986; the resulting investigations into safety standards may have caused the decrease.

**4. a.** **Step 1:** Record the data in a stem and leaf plot.

| Stem | Leaf |
|---|---|
| 4 | 9, 6 |
| 5 | 3, 6, 7, 8, 7 |
| 6 | 7, 8, 5, 4, 6, 3, 7, 0, 7, 0, 3, 4 |
| 7 | 3, 8, 9, 1, 4, 7, 0, 1, 2, 8 |
| 8 | 3, 5, 0, 8, 1 |
| 9 | 0, 0 |

**Step 2:** Arrange the numbers in each leaf in ascending order.

| Stem | Leaf |
|---|---|
| 4 | 6, 9 |
| 5 | 3, 6, 7, 7, 8 |
| 6 | 0, 0, 3, 3, 4, 4, 5, 6, 7, 7, 7, 8 |
| 7 | 0, 1, 1, 2, 3, 4, 7, 8, 8, 9 |
| 8 | 0, 1, 3, 5, 8 |
| 9 | 0, 0 |

**b.** J. Adams, Jefferson, Madison, J. Q Adams, Hoover, Truman, and Nixon died in their eighties and nineties.

**c.** Polk, Lincoln, Garfield, Arthur, McKinley, Harding, and Kennedy died in their forties and fifties.

**d.** These former presidents were assassinated.

## Section 2: Activity 1

**1.** **a.** very unlikely **b.** unlikely
**c.** likely **d.** very likely

**2.** **a.** There are three possible outcomes; the glass could land in each of these positions:

on its bottom on its top on its side

The set of possible outcomes is {on its bottom, on its top, on its side}.

**b.** The glass does not have a regular shape, so these outcomes will not be equally likely to occur.

**3.** **a.** The set of possible outcomes is {A, B, C, D, E, F}.

**b.** Yes, the cube has a regular shape, so the possible outcomes are equally likely to occur. (Assume that the cube is not unequally weighted.)

**4.** The spinner should twirl easily and not stick. Also the spinner should be twirled with enough force to ensure that it makes at least one complete revolution.

**5.** The set of possible outcomes is {A, B, C, D}.

**Note:** For each event, you can repeat this list of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes for each event.

**a.** The set of favourable outcomes is {A}.

$$P(\textit{vowel}) = \frac{1}{4} \quad \frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of the spinner landing on a vowel is $\frac{1}{4}$.

**b.** The set of favourable outcomes is {B, C, D}.

$$P(\textit{consonant}) = \frac{3}{4} \quad \frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of the spinner landing on a consonant is $\frac{3}{4}$.

**6.** The set of possible outcomes is $\{1, 2, 3, 4, 5, 6\}$.

**a.** The set of favourable outcomes is $\{1\}$.

$$P(1) = \frac{1}{6}$$

$$\frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of rolling a 1 is $\frac{1}{6}$.

**b.** The set of favourable outcomes is $\{2, 3, 5\}$.

$$P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$$

$$\frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of rolling a prime number is $\frac{1}{2}$.

**c.** The set of favourable outcomes is $\{4, 6\}$.

$$P(\text{composite}) = \frac{2}{6} = \frac{1}{3}$$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of rolling a composite number is $\frac{1}{3}$.

**7.** Let the first A be $A_1$.
Let the second A be $A_2$.

The set of possible outcomes is $\{D, A_1, T, A_2\}$.

**Note:** For each event, you can repeat this list of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes for each event.

**a.** The set of favourable outcomes is $\{D\}$.

$$P(D) = \frac{1}{4}$$

$$\frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of picking a tile labelled D is $\frac{1}{4}$.

**b.** The set of favourable outcomes is $\{A_1, A_2\}$.

$$P(A) = \frac{2}{4} = \frac{1}{2}$$

$$\frac{\text{Number of Favourable Outcomes}}{\text{Number of Possible Outcomes}}$$

The probability of picking a tile labelled A is $\frac{1}{2}$.

**c.** The set of favourable outcomes is $\{T\}$.

$$P(T) = \frac{1}{4}$$

Favourable Outcomes / Possible Outcomes

The probability of picking a tile labelled T is $\frac{1}{4}$.

**8.** Let one side of the coin be "Heads$_1$." Let the other side of the coin be "Heads$_2$."

The set of possible outcomes is $\{\text{Heads}_1, \text{Heads}_2\}$.

**a.** The set of favourable outcomes is $\{\text{Heads}_1, \text{Heads}_2\}$.

$$P(Heads) = \frac{2}{2} = 1$$

Favourable Outcomes / Possible Outcomes

The probability that the coin will land "Heads" up is 1.

A probability of 1 indicates a certain event.

**b.** The set of favourable outcomes is $\{\ \}$.

The set of $\{\ \}$ is read as "the empty set." There are no favourable outcomes.

$$P(Tails) = \frac{0}{2} = 0$$

Favourable Outcomes / Possible Outcomes

The probability that the coin will land "Tails" up is 0.

A probability of 0 indicates an impossible event.

**9.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| a vowel | 𝍸 𝍸 I | 11 |
| a consonant | 𝍸 𝍸 𝍸 𝍸 𝍸 IIII | 29 |

**a.** $P(vowel) = \frac{11}{40}$

Frequency of a Vowel / Total Number of Spins

The relative frequency of landing on a vowel in this experiment was $\frac{11}{40}$.

The theoretical probability of landing on a vowel is $\frac{1}{4}$. (See question 5.a. of this activity.)

Relative Frequency: $\frac{11}{40}$ Theoretical Probability: $\frac{1}{4} = \frac{10}{40}$

$\frac{11}{40} \doteq \frac{10}{40}$

In this experiment, the relative frequency of landing on a vowel was close to the theoretical probability.

**b.** $P(consonant) = \frac{29}{40}$ $\frac{\text{Frequency of a Consonant}}{\text{Total Number of Spins}}$

The relative frequency of landing on a consonant in this experiment was $\frac{29}{40}$.

The theoretical probability of landing on a consonant is $\frac{3}{4}$. (See question 5.b. of this activity.)

Relative Frequency: $\frac{29}{40}$ Theoretical Probability: $\frac{3}{4} = \frac{30}{40}$

$\frac{29}{40} \doteq \frac{30}{40}$

In this experiment, the relative frequency of landing on a consonant was close to the theoretical probability.

**10.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| 1 | 𝍸 \|\|\|\| | 9 |
| a prime number | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 \|\| | 32 |
| a composite number | 𝍸 𝍸 𝍸 \|\|\|\| | 19 |

**a.** $P(1) = \frac{9}{60}$ $\frac{\text{Frequency of 1}}{\text{Total Number of Rolls}}$

The relative frequency of rolling a 1 in this experiment was $\frac{9}{60}$.

The theoretical probability of rolling a 1 is $\frac{1}{6}$. (See question 6.a. of this activity.)

Relative Frequency: $\frac{9}{60}$ Theoretical Probability: $\frac{1}{6} = \frac{10}{60}$

$\frac{9}{60} \doteq \frac{10}{60}$

In this experiment, the relative frequency of rolling a 1 was close to the theoretical probability.

**b.** $P(\textit{prime}) = \frac{32}{60}$ $\left(\frac{\text{Frequency of a Prime}}{\text{Total Number of Rolls}}\right)$

The relative frequency of rolling a prime number in this experiment was $\frac{32}{60}$.

The theoretical probability of rolling a prime number is $\frac{1}{2}$. (See question 6.b. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{32}{60}$ | $\frac{1}{2} = \frac{30}{60}$ |

$\frac{32}{60} \doteq \frac{30}{60}$

In this experiment, the relative frequency of rolling a prime number was close to the theoretical probability.

**c.** $P(\textit{composite}) = \frac{19}{60}$ $\left(\frac{\text{Frequency of a Composite}}{\text{Total Number of Rolls}}\right)$

The relative frequency of rolling a composite number in this experiment was $\frac{19}{60}$.

The theoretical probability of rolling a composite number is $\frac{1}{3}$. (See question 6.c. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{19}{60}$ | $\frac{1}{3} = \frac{20}{60}$ |

$\frac{19}{60} \doteq \frac{20}{60}$

In this experiment, the relative frequency of rolling a composite number was close to the theoretical probability.

**11.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| D | 𝍸 III | 8 |
| A | 𝍸 𝍸 𝍸 𝍸 III | 23 |
| T | 𝍸 IIII | 9 |

**a.** $P(D) = \frac{8}{40}$ $\left(\frac{\text{Frequency of D}}{\text{Total Number of Picks}}\right)$

$= \frac{1}{5}$

The relative frequency of picking a tile labelled D in this experiment was $\frac{1}{5}$.

The theoretical probability of picking a tile labelled D is $\frac{1}{4}$. (See question 7.a. of this activity.)

Relative Frequency

$$\frac{1}{5} = \frac{8}{40}$$

Theoretical Probability

$$\frac{1}{4} = \frac{10}{40}$$

$$\frac{8}{40} \doteq \frac{10}{40}$$

In this experiment, the relative frequency of picking the letter D was close to the theoretical probability.

**b.** $P(A) = \frac{23}{40}$ $\left(\frac{\text{Frequency of A}}{\text{Total Number of Picks}}\right)$

The relative frequency of picking a tile labelled A in this experiment was $\frac{23}{40}$.

The theoretical probability of picking a tile labelled A is $\frac{1}{2}$. (See question 7.b. of this activity.)

Relative Frequency

$$\frac{23}{40}$$

Theoretical Probability

$$\frac{1}{2} = \frac{20}{40}$$

$$\frac{23}{40} \doteq \frac{20}{40}$$

In this experiment, the relative frequency of picking a tile labelled A was close to the theoretical probability.

**c.** $P(T) = \frac{9}{40}$ $\left(\frac{\text{Frequency of T}}{\text{Total Number of Picks}}\right)$

The relative frequency of picking a tile labelled T in this experiment was $\frac{9}{40}$.

The theoretical probability of picking a tile labelled T is $\frac{1}{4}$. (See question 7.c. of this activity.)

Relative Frequency

$$\frac{9}{40}$$

Theoretical Probability

$$\frac{1}{4} = \frac{10}{40}$$

$$\frac{9}{40} \doteq \frac{10}{40}$$

In this experiment, the relative frequency of picking a tile labelled T was close to the theoretical probability.

**12.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
|  | 𝍸 I | 6 |
|  | 𝍸 𝍸 𝍸 III | 18 |
|  | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I | 36 |

In this experiment, the relative frequency of the cup landing on its side was $\frac{36}{60}$ or $\frac{3}{5}$.

Based on the relative frequency in this experiment, the probability of the cup landing on its side is $\frac{3}{5}$.

**13.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
|  | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II | 42 |
|  | 𝍸 𝍸 𝍸 III | 18 |

In this experiment, the relative frequency of the tack landing with its point up was $\frac{42}{60}$ or $\frac{7}{10}$.

Based on the relative frequency in this experiment, the probability of the tack landing point up is $\frac{7}{10}$.

**14.** The Gryphons have won $\frac{4}{5}$ of the games against their rivals.

Based on this record, the probability that the Gryphons will win the next game against their rivals is $\frac{4}{5}$.

**15.** Dakota made 60% of his free throws.

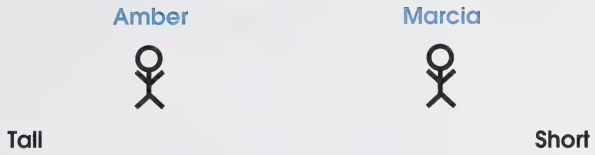
$$60\% = \frac{60}{100}$$
$$= \frac{3}{5}$$
$$\therefore\ P(\textit{free throw}) = \frac{3}{5}$$

Based on his past record, if Dakota is fouled in a game and rewarded a free throw, the probability that he will be successful is $\frac{3}{5}$.

## Now Try This

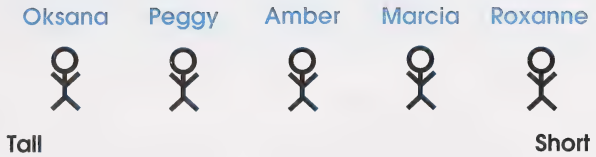
**16.** You can use diagrams and logic to help you solve this problem.

- Since Amber is taller than Marcia, the diagram looks like this.




- Since Peggy is taller than Amber and shorter than Oksana, the diagram now looks like this.


- Since Marcia is taller than Roxanne, the diagram now looks like this.

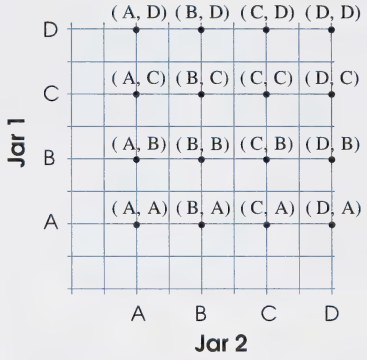



The students from tallest to shortest are Oksana, Peggy, Amber, Marcia, and Roxanne.

## Section 2: Activity 2

**1.** You may use graph paper or a table to list the possible outcomes.

### Graph Paper




### Table

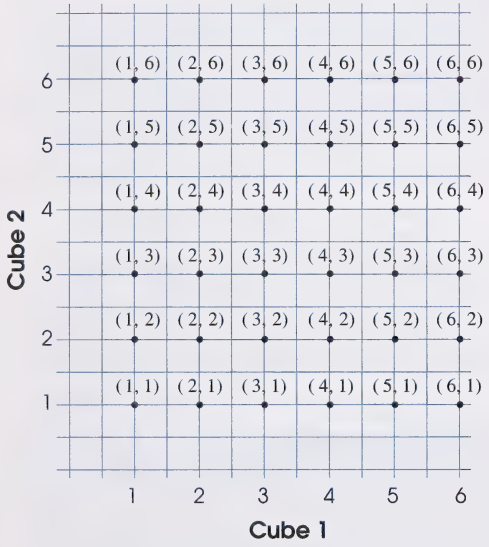
Jar 2

| Jar 1 | A | B | C | D |
|---|---|---|---|---|
| A | (A, A) | (A, B) | (A, C) | (A, D) |
| B | (B, A) | (B, B) | (B, C) | (B, D) |
| C | (C, A) | (C, B) | (C, C) | (C, D) |
| D | (D, A) | (D, B) | (D, C) | (D, D) |

The set of possible outcomes is $\{(A,A), (A,B), (A,C), (A,D), (B,A), (B,B), (B,C), (B,D), (C,A), (C,B), (C,C), (C,D), (D,A), (D,B), (D,C), (D,D)\}$.

**2.** You may use graph paper or a table to list the possible outcomes.

Graph Paper




Table

Cube 1

| Cube 2 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | (1, 1) | (1, 2) | (1, 3) | (1, 4) | (1, 5) | (1, 6) |
| 2 | (2, 1) | (2, 2) | (2, 3) | (2, 4) | (2, 5) | (2, 6) |
| 3 | (3, 1) | (3, 2) | (3, 3) | (3, 4) | (3, 5) | (3, 6) |
| 4 | (4, 1) | (4, 2) | (4, 3) | (4, 4) | (4, 5) | (4, 6) |
| 5 | (5, 1) | (5, 2) | (5, 3) | (5, 4) | (5, 5) | (5, 6) |
| 6 | (6, 1) | (6, 2) | (6, 3) | (6, 4) | (6, 5) | (6, 6) |

The set of possible outcomes is $\{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$.

**3.**

| First Spin | Second Spin | Possible Outcomes |
|---|---|---|
| A | A | (A, A) |
| | B | (A, B) |
| | C | (A, C) |
| | D | (A, D) |
| B | A | (B, A) |
| | B | (B, B) |
| | C | (B, C) |
| | D | (B, D) |
| C | A | (C, A) |
| | B | (C, B) |
| | C | (C, C) |
| | D | (C, D) |

This is the set of possible outcomes:

$$\{(A,A),\ (A,B),\ (A,C),\ (A,D),\ (B,A),\ (B,B),\ (B,C),\ (B,D),\ (C,A),\ (C,B),\ (C,C),\ (C,D)\}$$

**4.** Let H be "Heads". Let T be "Tails".

| Spin | Toss | Possible Outcomes |
|---|---|---|
| A | H | (A, H) |
| | T | (A, T) |
| B | H | (B, H) |
| | T | (B, T) |
| C | H | (C, H) |
| | T | (C, T) |

This is the set of possible outcomes:

$$\{(A,H),\ (A,T),\ (B,H),\ (B,T),\ (C,H),\ (C,T)\}$$

**5.** This is the set of possible outcomes.

$$\{(A,A),\ (A,B),\ (A,C),\ (A,D),\ (B,A),\ (B,B),\ (B,C),\ (B,D),\ (C,A),\ (C,B),\ (C,C),\ (C,D),\ (D,A),\ (D,B),\ (D,C),\ (D,D)\}$$

**Note:** For each event, you can repeat this list of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes for each event.

**a.** This is the set of favourable outcomes:

$$\{(A, A)\}$$

$$P(\textit{two vowels}) = \frac{1}{16}$$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of picking two vowels is $\frac{1}{16}$.

**b.** This is the set of favourable outcomes:

$$\{(B, B),\ (B, C),\ (B, D),\ (C, B),\ (C, C),\ (C, D),\ (D, B),\ (D, C),\ (D, D)\}$$

$$P(\textit{two consonants}) = \frac{9}{16}$$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of picking two consonants is $\frac{9}{16}$.

**c.** This set of favourable outcomes:

$$\{(A, B),\ (A, C),\ (A, D),\ (B, A),\ (C, A),\ (D, A)\}$$

$$P(\textit{vowel and consonant}) = \frac{6}{16} = \frac{3}{8}$$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of picking a vowel and a consonant is $\frac{3}{8}$.

**6.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| two vowels | //// | 4 |
| two consonants | 𝍸 𝍸 𝍸 𝍸 𝍸 | 25 |
| vowel and consonant | 𝍸 𝍸 𝍸 //// | 19 |

**a.** $P(\textit{two vowels}) = \frac{4}{48} = \frac{1}{12}$

$$\frac{\text{Frequency of Two Vowels}}{\text{Total Number of Picks}}$$

The relative frequency of picking two vowels in this experiment was $\frac{1}{12}$.

The theoretical probability of picking two vowels is $\frac{1}{16}$. (See question 5.a. of this activity.)

Relative Frequency: $\frac{1}{12} = \frac{4}{48}$

Theoretical Probability: $\frac{1}{16} = \frac{3}{48}$

$$\frac{4}{48} \doteq \frac{3}{48}$$

In this experiment, the relative frequency of picking two vowels was close to the theoretical probability.

**b.** $P(\textit{two consonants}) = \frac{25}{48}$

$$\frac{\text{Two Consonants}}{\text{Total Number of Picks}}$$

The relative frequency of picking two consonants in this experiment was $\frac{25}{48}$.

The theoretical probability of picking two consonants is $\frac{9}{16}$. (See question 5.b. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{25}{48}$ | $\frac{9}{16} = \frac{27}{48}$ |

$$\frac{25}{48} \doteq \frac{27}{48}$$

In this experiment, the relative frequency of picking two consonants was close to the theoretical probability.

**c.** $P(\textit{vowel and consonant}) = \frac{19}{48}$

$$\frac{\text{Vowel and Consonant}}{\text{Total Number of Picks}}$$

The relative frequency of picking a vowel and a consonant in this experiment was $\frac{19}{48}$.

The theoretical probability of picking a vowel and a consonant is $\frac{3}{8}$. (See question 5.c. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{19}{48}$ | $\frac{3}{8} = \frac{18}{48}$ |

$$\frac{19}{48} \doteq \frac{18}{48}$$

In this experiment, the relative frequency of picking a vowel and a consonant was close to the theoretical probability.

**7.** Let H be "Heads." Let T be "Tails."

This is the set of possible outcomes:

$$\{(A,H),\ (A,T),\ (B,H),\ (B,T),\ (C,H),\ (C,T)\}$$

**Note:** For each event, you can repeat this list of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes for each event.

**a.** This is the set of favourable outcomes:

$$\{(A,H)\}$$

$P(\textit{vowel and Heads}) = \frac{1}{6}$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of landing on a vowel and then tossing "Heads" is $\frac{1}{6}$.

**b.** This is the set of favourable outcomes:

$$\{(B, H),\ (C, H)\}$$

$$P(\textit{consonant and Heads}) = \frac{2}{6} = \frac{1}{3}$$

$$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$

The probability of landing on a consonant and then tossing "Heads" is $\frac{1}{3}$.

**8.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| a vowel and "Heads" | 𝍸 III | 8 |
| a consonant and "Heads" | 𝍸 𝍸 I | 11 |
| a vowel and "Tails" | 𝍸 II | 7 |
| a consonant and "Tails" | 𝍸 𝍸 | 10 |

**a.** $P(\textit{vowel and Heads}) = \frac{8}{36} = \frac{2}{9}$

$$\frac{\text{Vowel and "Heads"}}{\text{Total Number of Trials}}$$

The relative frequency of a vowel and "Heads" occurring in this experiment was $\frac{2}{9}$.

The theoretical probability of a vowel and "Heads" occurring is $\frac{1}{6}$. (See question 7.a. of this activity.)

Relative Frequency $\quad\quad$ Theoretical Probability

$$\frac{2}{9} = \frac{8}{36} \qquad \frac{1}{6} = \frac{6}{36}$$

$$\frac{8}{36} \div \frac{6}{36}$$

In this experiment, the relative frequency was close to the theoretical probability.

**b.** $P(\textit{consonant and Heads}) = \frac{11}{36}$

$$\frac{\text{Consonant and "Heads"}}{\text{Total Number of Trials}}$$

The relative frequency of a consonant and "Heads" occurring in this experiment was $\frac{11}{36}$.

The theoretical probability of a consonant and "Heads" occurring is $\frac{1}{3}$. (See question 7.b. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{11}{36}$ | $\frac{1}{4} = \frac{9}{36}$ |

$\frac{11}{36} \doteq \frac{9}{36}$

In this experiment, the relative frequency was close to the theoretical probability.

**9.** This is the set of possible outcomes:

$$\{(A,A),\ (A,B),\ (A,C),\ (A,D),\ (B,A),\ (B,B),\ (B,C),\ (B,D),\ (C,A),\ (C,B),\ (C,C),\ (C,D)\}$$

**Note:** For each event, you can repeat this list of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes for each event.

**a.** The set of favourable outcomes is $\{(A,A)\}$.

$$P(\textit{two vowels}) = \frac{1}{12}$$

$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$

The probability of landing on two vowels is $\frac{1}{12}$.

**b.** This is the set of favourable outcomes:

$$\{(B,B),\ (B,C),\ (B,D),\ (C,B),\ (C,C),\ (C,D)\}$$

$$P(\textit{two consonants}) = \frac{6}{12} = \frac{1}{2}$$

$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$

The probability of landing on two consonants is $\frac{1}{2}$.

**c.** This is the set of favourable outcomes:

$$\{(A,B),\ (A,C),\ (B,A),\ (C,A)\}$$

$$P(\textit{vowel and consonant}) = \frac{4}{12} = \frac{1}{3}$$

$\frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$

The probability of landing on a vowel and a consonant is $\frac{1}{3}$.

**10.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| two vowels | //// | 4 |
| two consonants | 𝍸 𝍸 𝍸 // | 17 |
| a vowel and a consonant | 𝍸 𝍸 𝍸 | 15 |

**a.** $P(\textit{two vowels}) = \frac{4}{36}$ $\left(\frac{\text{Two Vowels}}{\text{Total Number of Trials}}\right)$

$= \frac{1}{9}$

The relative frequency of two vowels occurring in this experiment was $\frac{1}{9}$.

The theoretical probability of two vowels occurring is $\frac{1}{12}$. (See question 9.a. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{1}{9} = \frac{4}{36}$ | $\frac{1}{12} = \frac{3}{36}$ |

$\frac{4}{36} \doteq \frac{3}{36}$

In this experiment, the relative frequency was close to the theoretical probability.

**b.** $P(\textit{two consonants}) = \frac{17}{36}$ $\left(\frac{\text{Two Consonants}}{\text{Total Number of Trials}}\right)$

The relative frequency of two consonants occurring in this experiment was $\frac{17}{36}$.

The theoretical probability of two consonants occurring is $\frac{1}{2}$. (See question 9.b. of this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{17}{36}$ | $\frac{1}{2} = \frac{18}{36}$ |

$\frac{17}{36} \doteq \frac{18}{36}$

In this experiment, the relative frequency was close to the theoretical probability.

**c.** $P(\textit{vowel and consonant}) = \frac{15}{36}$ $\left(\frac{\text{Vowel and Consonant}}{\text{Number of Trials}}\right)$

$= \frac{5}{12}$

The relative frequency of a vowel and a consonant occurring in this experiment was $\frac{15}{36}$.

The theoretical probability of a vowel and a consonant occurring is $\frac{1}{3}$. (See question 9.c. in this activity.)

| Relative Frequency | Theoretical Probability |
|---|---|
| $\frac{5}{12}$ | $\frac{1}{3} = \frac{4}{12}$ |

$\frac{5}{12} \doteq \frac{4}{12}$

In this experiment, the relative frequency was close to the theoretical probability.

**11.** This is the set of possible outcomes:

$$\{(1,1),\ (1,2),\ (1,3),\ (1,4),\ (1,5),\ (1,6),\ (2,1),\ (2,2),\ (2,3),\ (2,4),\ (2,5),\ (2,6),\ (3,1),\ (3,2),\ (3,3),\ (3,4),\ (3,5),\ (3,6),\ (4,1),\ (4,2),\ (4,3),\ (4,4),\ (4,5),\ (4,6),\ (5,1),\ (5,2),\ (5,3),\ (5,4),\ (5,5),\ (5,6),\ (6,1),\ (6,2),\ (6,3),\ (6,4),\ (6,5),\ (6,6)\}.$$

**Note:** For each event, you can list the set of possible outcomes and circle the favourable outcomes. However, many students prefer to list the set of possible outcomes **once** and then mentally identify the favourable outcomes.

**a.** This is the set of favourable outcomes:

$$\{(1,1),\ (2,2),\ (3,3),\ (4,4),\ (5,5),\ (6,6)\}$$

$$P(\textit{two identical numbers}) = \frac{6}{36} \quad \frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$
$$= \frac{1}{6}$$

The probability of rolling two identical numbers is $\frac{1}{6}$.

**b.** This is the set of favourable outcomes:

$$\{(1,6),\ (2,5),\ (3,4),\ (4,3),\ (5,2),\ (6,1)\}$$

$$P(\textit{sum of } 7) = \frac{6}{36} \quad \frac{\text{Favourable Outcomes}}{\text{Possible Outcomes}}$$
$$= \frac{1}{6}$$

The probability of rolling two numbers with a sum of 7 is $\frac{1}{6}$.

**12.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| two identical numbers | 𝍸 \| | 8 |
| two numbers with a sum of 7 | 𝍸 | 5 |
| other | 𝍸 𝍸 𝍸 𝍸 \| | 23 |

**a.** $P(\textit{two identical numbers}) = \frac{8}{36} \quad \frac{\text{Two Identical Numbers}}{\text{Total Number of Rolls}}$
$= \frac{2}{9}$

The relative frequency of rolling two identical numbers in this experiment was $\frac{2}{9}$.

The theoretical probability of rolling two identical numbers is $\frac{1}{6}$. (See question 11.a. of this activity.)

Relative Frequency: $\frac{2}{9} = \frac{8}{36}$ Theoretical Probability: $\frac{1}{6} = \frac{6}{36}$

$\frac{8}{36} \doteq \frac{6}{36}$

In this experiment, the relative frequency was close to the theoretical probability.

**b.** $P(\textit{sum of } 7) = \frac{5}{36}$ $\left(\frac{\text{Frequency of a Sum of 7}}{\text{Total Number of Rolls}}\right)$

The relative frequency of rolling two numbers with a sum of 7 in this experiment was $\frac{5}{36}$.

The theoretical probability of rolling two numbers with a sum of 7 is $\frac{1}{6}$. (See question 11.b. of this activity.)

Relative Frequency: $\frac{5}{36}$ Theoretical Probability: $\frac{1}{6} = \frac{6}{36}$

$\frac{5}{36} \doteq \frac{6}{36}$

In this experiment, the relative frequency was close to the theoretical probability.

**13.** Answers will vary. This is one student's experimental results and answers.

**a.**

| Event | Tally | Frequency |
|---|---|---|
| Heads | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 II | 47 |
| Tails | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III | 53 |

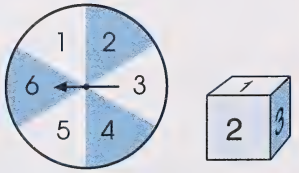
Based on the simulation, Thor will get about 47 questions right.

**b.**

| Event | Tally | Frequency |
|---|---|---|
| C | 𝍸 𝍸 𝍸 𝍸 IIII | 24 |
| W | 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 I | 76 |

Based on the simulation, Jessica will get about 24 questions right.

**14. a.** To model this situation, you can use either of these probability tools. Let each sector of the dial or each face of the cube represent a different baseball card.



Follow these steps.

**Step 1:** Make a frequency table like this.

| Event | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |

**Step 2:** Twirl the spinner on the dial or roll the cube and record the results in the frequency table.

Repeat the action until you have a tally mark beside each of the numbers.

**Step 3:** Count the total number of tally marks. This is an estimate of the number of boxes of cereal you would need to purchase.

**b.** Answers will vary. This is one student's experimental results and answers.

| Event | Tally | Frequency |
|---|---|---|
| 1 | /// | 3 |
| 2 | // | 2 |
| 3 | //// | 4 |
| 4 | /// | 3 |
| 5 | // | 2 |
| 6 | / | 1 |

The total number of rolls is 15.

In this experiment, 15 rolls were required to obtain at least one of each number.

So, you will need to purchase about 15 boxes of cereal to obtain one of each baseball card.

## Now Try This

**15.** Use the guess, check, and revise strategy to solve this problem. Answers may vary. Here is one possibility.

| | | | | | |
|---|---|---|---|---|---|
| X | X | X | X | X | X |
| X | X | X | | X | |
| | X | X | | X | X |
| | X | X | X | X | |

## Section 2: Activity 3

**1. a.** Census takers drop off questionnaires at every household in Canada before the census day. They return after the census day and collect the questionnaires. The results are entered into computers.

**b.** A census is conducted every five years.

**c.** Answers will vary depending on the year researched. In 1991, the Canadian census cost over $250 000 000.

**d.** The census provides data that are used to determine electoral boundaries, federal payments to the provinces, and provincial payments to cities. Data are also used to formulate social and economic policies.

**2.** Answers will vary. This is one student's survey results.

| Number | Tally | Frequency |
|---|---|---|
| 1 | / | 1 |
| 2 | // | 2 |
| 3 | 𝍸 / | 6 |
| 4 | / | 1 |

The following answers are based on these survey results.

**a.**

| Number | Tally | Frequency | Percent Frequency |
|---|---|---|---|
| 1 | / | 1 | 10% |
| 2 | // | 2 | 20% |
| 3 | 𝍸 / | 6 | 60% |
| 4 | / | 1 | 10% |

**Note:** Here is how the percent frequencies were calculated.

$$\frac{1}{10} = \frac{10}{100} = 10\% \qquad \frac{2}{10} = \frac{20}{100} = 20\% \qquad \frac{6}{10} = \frac{60}{100} = 60\%$$

**b.** Based on the survey results, the next person will likely select 3.

**c.** The results may surprise you. You would assume that the numbers would be equally chosen. For some reason, people do not choose the numbers randomly; they prefer 3.

**3.** Answers will vary. This is one student's results.

| Letter | Tally | Frequency |
|---|---|---|
| a | 𝍸 \|\| | 7 |
| b | \|\| | 2 |
| c | \|\| | 2 |
| d | \|\| | 2 |
| e | 𝍸 𝍸 \|\| | 12 |
| f | 𝍸 \| | 6 |
| g | \| | 1 |
| h | 𝍸 \|\| | 7 |
| i | \|\|\|\| | 4 |
| j | | 0 |
| k | \| | 1 |
| l | 𝍸 | 5 |
| m | \| | 1 |
| n | 𝍸 \|\|\| | 8 |
| o | 𝍸 \|\|\|\| | 9 |
| p | \| | 1 |
| q | \| | 1 |
| r | 𝍸 \|\| | 7 |
| s | 𝍸 \| | 6 |
| t | 𝍸 𝍸 | 10 |
| u | 𝍸 \|\| | 7 |
| v | | 0 |
| w | \|\|\| | 3 |
| x | | 0 |
| y | | 0 |
| z | | 0 |

The following answers are based on these survey results.

**a.** Based on the survey, the two most frequently used vowels are *e* and *o*.

**b.** Based on the survey, the two most frequently used consonants are *t* and *n*.

**4.** **Step 1:** Calculate how many degrees of the circle graph each sport should be. **Hint:** There were 30 people surveyed. There are 360° in a circle.

**Basketball**

$$\frac{10}{30} \xrightarrow{\times 12} \frac{120}{360}$$

**Football**

$$\frac{5}{30} \xrightarrow{\times 12} \frac{60}{360}$$

**Hockey**

$$\frac{7}{30} \xrightarrow{\times 12} \frac{84}{360}$$

**Soccer**

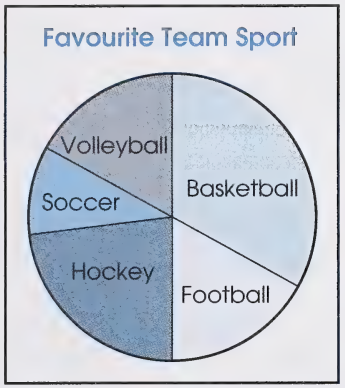
$$\frac{3}{30} \xrightarrow{\times 12} \frac{36}{360}$$

**Volleyball**

$$\frac{5}{30} \xrightarrow{\times 12} \frac{60}{360}$$

**Step 2:** Use a compass, straightedge, and protractor to draw the circle graph.

Basketball will be represented by 120°, football by 60°, hockey by 84°, soccer by 36°, and volleyball by 60°. Label each sector and give the graph a suitable title.




The total number of degrees of the sectors in a circle graph should equal 360°.

**5. Step 1:** Calculate how many degrees of the circle graph each ice-cream flavour should be. **Hint:** There was a total of 40 people surveyed. There are 360° in a circle.

**Bubble Gum**

$$\frac{9}{40} \overset{\times 9}{=} \frac{81}{360}$$

**Chocolate**

$$\frac{15}{40} \overset{\times 9}{=} \frac{135}{360}$$

**Maple Walnut**

$$\frac{2}{40} \overset{\times 9}{=} \frac{18}{360}$$

**Strawberry**

$$\frac{8}{40} \overset{\times 9}{=} \frac{72}{360}$$

**Vanilla**

$$\frac{6}{40} \overset{\times 9}{=} \frac{54}{360}$$

**Step 2:** Use a compass, straightedge, and protractor to draw the circle graph. Bubble gum will be represented by 81°, chocolate by 135°, maple walnut by 18°, strawberry by 72°, and vanilla by 54°.

**6.** Answers will vary.

You may prefer written surveys because you can take your time and think about your answers. On the other hand, you may not enjoy reading and would prefer to have the questionnaire read to you.

**7.** Telephone interviewers and personal interviewers need to be trained so that they do not influence the survey results by stressing a particular word or by giving the respondents extra information.

**8.** **a.** Maxine's Restaurant probably knows that people feel uncomfortable giving their names to companies; it is an invasion of their privacy. Customers are more inclined to complete these types of surveys anonymously.

**b.** The Alberta Distance Learning Centre keeps the results of the course surveys confidential, but asks for your name, address, and age so that you can be contacted personally if you have serious concerns or if further clarification should be required.

**9.** **a.** A multiple-choice questionnaire is used.

**b.** The ADLC questionnaire uses yes/no questions, multiple-choice questions, written responses, and numerical responses.

**10.** Answers will vary.

You may feel the question is too difficult to understand because so many facts have been listed. On the other hand, you may feel that all the information is needed.

**11.** **a.** Choice B is better.

The meaning of the word *large* in Choice A is unclear; you may interpret the word *large* in different ways.

**b.** Choice B is better.

You may be confused by the word *not* in Choice A.

**12.** **a.** Choice B is better.

Choice A is attempting to influence you with the word *reckless*.

**b.** Choice A is better.

Choice B is attempting to influence you by using the words *intelligent* and *hardworking*.

**13.** **a.** Paul and Sita LaGrange Rao considered not answering this question because they felt people should be classified by citizenship rather than by race.

**b.** Answers will vary. The government may have wanted to obtain as accurate information as possible about the diversity of Canada's population.

**Note:** Find the article entitled "Mandatory Race Question in Census Defended" in the Appendix. Read the article to discover Canada's assistant chief statistician's explanation for asking the question.

## Now Try This

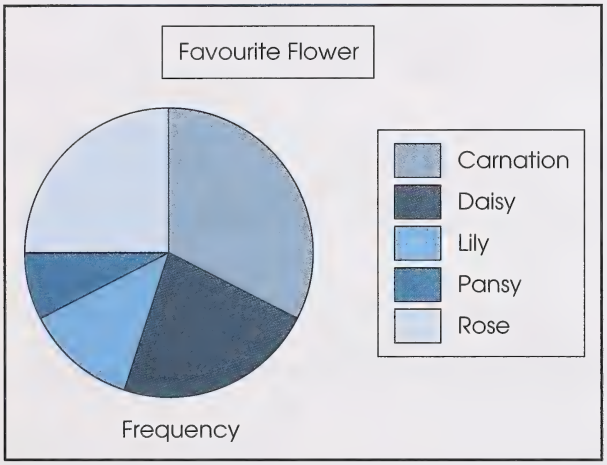
**14.** You can simulate the problem using 25 objects which have a front and a back such as coins or playing cards. (Begin with all the objects front up to represent the closed lockers which the instructor saw. Flip the objects as required to solve the problem.)

Twenty locker doors are closed. (The first, fourth, ninth, sixteenth, and twenty-fifth locker doors are open.)

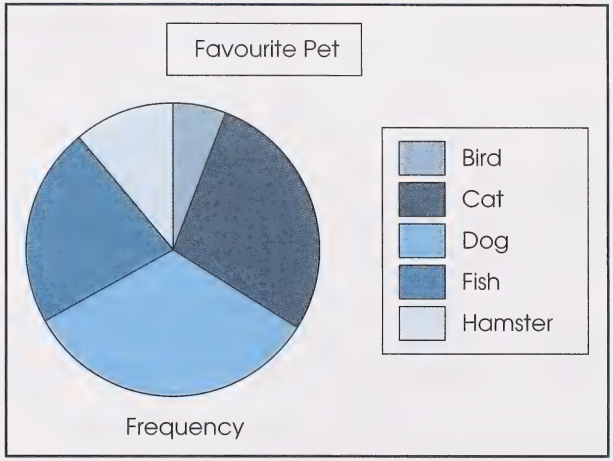
# Section 2: Follow-up Activities

## Extra Help

**1.**




**2.**




## Enrichment

**1. a.** Let 1 be January, 2 be February,..., 12 be December. Make a frequency table like this. Then follow the steps using Method 1 or Method 2.

| Event | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |

### Method 1: Using a Computer

This answer uses the *ClarisWorks*™ spreadsheet.

**Step 1:** In a spreadsheet, click on Cell A1, enter the formula =RAND(12), and press "Return."

**Step 2:** Record the result in the frequency table.

**Step 3:** Obtain a second number by clicking on "Calculate" in the menu bar and dragging down to "Calculate Now."

**Step 4:** Repeat steps 2 and 3 until there is at least one tally mark beside each outcome.

**Step 5:** Count the number of trials. This is the number of people that you would need to gather together.

### Method 2: Using Tiles of Paper and a Bag

**Step 1:** Gather 12 identical tiles. Label the tiles 1 through 12. Put the tiles in a bag.

**Step 2:** Mix the tiles thoroughly and, without looking, pick a tile. Record the result in the frequency table. Replace the tile.

**Step 3:** Repeat step 2 until there is at least one tally mark beside each outcome.

**Step 4:** Count the number of trials. This is the minimum number of people required.

**b.** Answers will vary. This is one student's response based on a simulation.

You should pick about 30 people.

**2. a.** Let 1 be January 1, 2 be January 2,..., 32 be February 1,..., 60 be February 29,..., and 366 be December 31. Make a frequency table like this. Then follow the steps in Method 1 or Method 2.

| Event | Tally | Frequency |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |

### Method 1: Using a Computer

This answer uses the *ClarisWorks*™ spreadsheet.

**Step 1:** In a spreadsheet, click on Cell A1, enter the formula =RAND(366), and press "Return."

**Step 2:** Record the result in the frequency table.

**Step 3:** Obtain a second number by clicking on "Calculate" in the menu bar and dragging down to "Calculate Now."

**Step 4:** Repeat steps 2 and 3 until two identical outcomes occur.

**Step 5:** Count the number of trials. This is the number of people you should pick.

### Method 2: Using Tiles of Paper and a Bag

**Step 1:** Gather 366 identical tiles of paper. Label the tiles 1 through 366. Put the tiles in a bag.

**Step 2:** Mix the tiles thoroughly and, without looking, pick a tile. Record the result in the frequency table. Replace the tile.

**Step 3:** Repeat Step 3 until two identical outcomes occur.

**Step 4:** Count the number of trials. This is the number of people you should pick.

**b.** Answers will vary. This is one student's response based on a simulation.

You should pick about 32 people.

# 'Plaid' Family Would Rather Skip Question[1]

## WHERE DO YOU FIT IN?

A question on racial origins will be submitted to the one in five Canadian households that receive the detailed census May 14. People will be asked the following question:

"Is this person: (Mark or specify more than one, if applicable)

- ☐ White
- ☐ Chinese
- ☐ South Asian (e.g., East Indian, Pakistani, Punjabi, Sri Lankan)
- ☐ Black (e.g., African, Haitian, Jamaican, Somali)
- ☐ Arab/West Asian (e.g., Armenian, Egyptian, Iranian, Lebanese, Moroccan)
- ☐ Filipino
- ☐ South East Asian (e.g., Cambodian, Indonesian, Laotian, Vietnamese)
- ☐ Latin American
- ☐ Japan
- ☐ Korean
- ☐ Other—Specify"

There are separate questions for aboriginal Canadians—defined as North American Indians, Metis, or Inuit—who will also be asked about their band and treaty status.

Go ahead and fine him the $500, says Paul LaGrange Rao. If he's among the one in five Canadians asked to provide his race in the May census, he's not answering.

"I think it's ridiculous," he says. "I don't think the government needs to know that information. If you're a citizen, you're a citizen. That's all that matters."

Paul, 28, and his wife, Sita, 24, own and operate the Second Cup coffee shop on Whyte Avenue. He's also part-time fund-raising consultant. She studies French literature at the University of Alberta. They're also the parents of a 20-month-old daughter, Maya. And all those things, they say, tell people more about them than their respective races.

"Colour is only one distinguishing characteristic," says Sita. "On my mother's side, I'm German, Scottish, and English. But my father comes from India. People are always asking me where I'm from, and it kind of bugs me that it's not good enough to say that you're Canadian."

For his part, Paul says, his mother's family arrived in Quebec "sometime before the Battle of the Plains of Abraham." His father's relatives arrived in Canada from Belgium and England in the 19th century.

"We're as Canadian as you can get," he says. And that, he says, is all Statistics Canada should need to know. The internment of Japanese-Canadians during the Second World War, he argues, shows what could happen when a government classifies people by race, not citizenship.

"It just deepens the divisions in our society," he says. "And what does our daughter fill in? Is there a plaid box for her?"

"I'm totally mixed, and so is Maya," says Sita. If she does get one of the controversial census forms, she'll check off two boxes, one for white, and one for South Asian. But she won't be happy about it.

"I can understand why they're asking, for research, for statistical purposes. I've used census data to do research myself. But I hope that I don't get asked.

---

[1] *The Edmonton Journal* for the article by Paula Simons from January 14, 1996, page E1. Reprinted by permission of *The Edmonton Journal*.

# Mandatory Race Question in Census Defended[1]

Canada's assistant chief statistician defends the decision to question Canadians on racial background in this year's national census, saying it was neither divisive nor racist.

Bruce Petrie said Wednesday information from the census will provide knowledge and understanding on Canada's diverse population. "It's only divisive if people misuse the information or feel it's a threat."

The race question will face one in five Canadians who will get the long version of the 1996 census on May 14. It asks people to specify their racial origins by checking off one of 10 boxes that include: South Asian, Arab, black and white. The categories are defined by federal Employment Equity legislation.

Participation is mandatory. All Canadians are required by law to fill out the census and those who refuse will be passed on to the Department of Justice. Petrie said about 20 cases went to court from the last census.

Being asked to identify their racial backgrounds has not been well-received by many Canadians. Reform MP Mike Scott wants people to write in "Martian" when asked about race. Letter writers to *The Journal* have gone as far as comparing it to Nazi tactics.

But Petrie said the question is designed only to enumerate the visible minority population for the federal government to comply with equity legislation to promote equal employment opportunities for women, aboriginal peoples, the disabled and members of visible minorities.

In a speech to the Canadian Club of Edmonton, Petrie said the question was carefully designed and tested. And the Canadian Human Rights Commission calls the question reasonable in a society which needs to understand itself.

"Some of the criticism appears to be directed more at the idea of employment equity than the collection of statistics," Petrie said.

Since 1986, estimates of Canada's visible minority population have been derived from a question on ethnic and cultural origins, but the large numbers of people identifying themselves as "Canadian" provided no indication of whether they were a member of a visible minority.

## A SENSE OF CENSUS

- May 14 is national census day in Canada.
- Most of us will get a short questionnaire with only seven questions, but one in five will get a long version that should take about half an hour to fill out. In the Northwest Territories, everyone will have to fill out a long questionnaire.
- The question about ethnic background is only on the long version.
- In Alberta, 4,500 people will be employed by this year's census. In Edmonton, about 1,250 will be hired.
- The budget for the Alberta portion of the census is $12.8 million.
- $5 million is budgeted for collection of the census forms in Edmonton alone.

[1] *The Edmonton Journal* for the article by James Stevenson from April 18, 1996, page A3. Reprinted by permission of *The Edmonton Journal*.

# COURSE SURVEY FOR MATHEMATICS 7 (1996)

***After you have completed the assignments in this course, please fill out this questionnaire and mail it to the address given on the last page. This course is designed in a new distance learning format, so we are interested in your responses. Your constructive comments will be greatly appreciated, as future course revisions can then incorporate any necessary improvements.***

Name _____ Age ❑ under 19 ❑ 19 to 40 ❑ over 40

Address _____ File No. _____

_____ Date _____

_____

## Design

1. This course contains a series of Student Module Booklets. Do you like the idea of separate booklets?

_____

_____

2. Have you ever enrolled in a correspondence or distance learning course that arrived as one large volume?

❑ Yes ❑ No If yes, which style do you prefer?

_____

_____

3. The Student Module Booklets contain a variety of self-assessed activities. Did you find it helpful to be able to check your work and have immediate feedback?

❑ Yes ❑ No If yes, explain.

_____

_____

4. Were the questions and directions easy to understand?

❑ Yes ❑ No If no, explain.

_____

_____

5. Each section contains follow-up activities. Which type of follow-up activity did you choose?

   ❑ mainly extra help
   ❑ mainly enrichment
   ❑ a variety
   ❑ none

   Did you find these activities beneficial?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

6. Did you understand what was expected in the Assignment Booklets?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

7. The course materials were designed to be completed by students working independently at a distance. Were you always aware of what you had to do?

   ❑ Yes ❑ No If no, provide details.

   _____

   _____

8. This distance learning course may include an assortment of drawings, photographs, and charts.

   a. Did you find the visuals in this course helpful?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

   b. Did you find the variety of visuals in this course motivating?

      ❑ Yes ❑ No Comment on the lines below.

      _____

      _____

9. Suggestions for audiocassette, videocassette, and computer activities may have been included in the course. Did you complete these media activities?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

**Only students enrolled in a Junior High course need to complete the following question.**

10. The Student Module Booklet may have directed you to work with your learning facilitator. How well did you work as a team?

    Student's comments: _____

    _____

    _____

    _____

    Learning Facilitator's comments: _____

    _____

    _____

    _____

## Course Content

1. Was enough detailed information provided to help you learn the expected skills and ojectives?

   ❑ Yes ❑ No Comment on the lines below.

   _____

   _____

2. Did you find the work load reasonable?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

3. Did you have any difficulty with the reading level?

❑ Yes ❑ No Please comment.

_____

_____

4. How would you assess your general reading level?

❑ poor reader ❑ average reader ❑ good reader

5. Was the material presented clearly and with sufficient depth?

❑ Yes ❑ No If no, explain.

_____

_____

## General

1. What did you like least about the course?

_____

_____

2. What did you like most about the course?

_____

_____

## Additional Comments

_____

_____

_____

_____

_____

_____

_____

**Only students enrolled with the Alberta Distance Learning Centre need to complete the remaining questions.**

1. Did you contact the Alberta Distance Learning Centre for help or information while doing your course?

   ❑ Yes ❑ No If yes, approximately how many times? _____

   Did you find the staff helpful?

   ❑ Yes ❑ No If no, explain.

   _____

   _____

2. Were you able to fax any of your assignment response pages?

   ❑ Yes ❑ No If yes, comment on the value of being able to do this.

   _____

   _____

3. If you mailed your assignment response pages, how long did it take for their return?

   _____

   _____

4. Was the feedback you received from your correspondence or distance learning teacher helpful?

   ❑ Yes ❑ No Please comment.

   _____

   _____

   _____

   _____

Thanks for taking the time to complete this questionnaire. Your feedback is important to us. Please return this questionnaire to the address on the right.

If you are enrolled at the Alberta Distance Learning Centre and have been mailing your Assignment Booklets to ADLC, you may return this questionnaire with the final Assignment Booklet in the course.

Instructional Design and Development
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LRDC
Producer

Mathematics 7
Student Module Booklet
Module 7

1996